Radiative instability of open beam-plasma systems

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A new expression is derived for the growth rate of the instability of an open beam-plasma system as a function of the density of the ultrarelativistic electron beam.

With increasing density of an ultrarelativistic electron beam in a transversely bounded plasma, several new phenomena come into play[1] and affect the course of the Čerenkov instability: The behavior of the growth rate as a function of the frequency and parameters of the beam-plasma system changes, and the polarization and the transverse structure of the electromagnetic waves excited by the beam are distorted.[2–5]

In the present letter we show that at large values of the interaction parameter $\nu = \gamma(n_b/n_p) \gg 1$ (γ is the relativistic factor of the beam electrons, and n_b and n_p are the densities of the beam and the plasma) an instability occurs in an open beam-plasma system. This instability is accompanied by a significant emission of electromagnetic energy from the lateral boundaries of the plasma. The functional dependence of the maximum spatial growth rate on the beam density is distinct: $\propto n_b^{3/4}$. None of the known functional dependences[2–6] reduces to this one.

For our analysis we adopt the example of a two-dimensional model of a plane beam-plasma slab of thickness d in the region $-(d/2) < x < (d/2)$, $-\infty \leqslant y,z \leqslant \infty$. The z axis runs along the direction of the beam. All of the beam electrons are in translational motion with an unperturbed velocity u; the relativistic factor satisfies $\gamma = (1 - u^2/c^2)^{-1/2} \gg 1$. The entire system is immersed in a strong magnetic field, which prevents transverse motion of the electrons. We ignore the motion of the ions and the thermal velocity of the electrons.

The continuity conditions on the fields and the derivatives at the boundaries of the slab give us a dispersion relation which links the frequency ω and the longitudinal wave number k of the electromagnetic perturbations which have a nonzero longitudinal component of the electric field and which vary outside the slab in accordance with $\exp(-i\omega t + ikz - \kappa|x|$:

$$(1 - \sqrt{\epsilon_\parallel})^2/(1 + \sqrt{\epsilon_\parallel})^2 = \exp(2\kappa\sqrt{\epsilon_\parallel} d), \qquad (1)$$

where $\kappa = (k^2 - \omega^2/c^2)^{1/2}$ is the transverse wave number, $\epsilon_\parallel = 1 - \omega_p^2/\omega^2 - \omega_b^2/[\gamma^3(\omega - ku)^2]$ is the longitudinal dielectric constant of the slab, $\omega_{p,b} = (4\pi e^2 n_{p,b}/m)^{1/2}$ are the plasma frequencies of the plasma and the beam, and e and m are the charge and mass of an electron.

It follows from an analysis of Eq. (1) that at sufficiently large values of γ, satisfying the conditions

$$\omega_p d/c \ll \gamma, \tag{2}$$

$$\omega_b^2 \omega_p^2 d^4/c^4 \ll \gamma^3, \tag{3}$$

the beam can interact with only a single plasma mode,[1)] whose amplitude varies slowly inside the slab ($|\kappa\sqrt{\epsilon_\parallel}|d \ll 1$). The length scale of the penetration of the electromagnetic field into vacuum is considerably greater than the thickness d. At small values of the interaction parameter ($\nu^{1/3} \ll 1$) we have the "ordinary" Čerenkov instability[6] with a maximum spatial growth rate $|\mathrm{Im}k|_m = (\sqrt{3}/4)(\omega_p^2 d/c^2\gamma^3)\nu^{1/3} \propto n_b^{1/3}$, which is reached at the frequency $\omega_0 = (\omega_p^2 d/2c\gamma) \ll \omega_p$. The frequency ω_0 corresponds to the precise equality of the beam velocity and the phase velocity of the plasma mode: $u = \omega_0/k_p(\omega_0)$, where $k_p(\omega) \simeq (\omega/c)(1 + (2\omega^2c^2/\omega_p^4 d^2))$ is the dispersion relation $k(\omega)$ in the absence of the beam. Here $\kappa^{-1} \simeq c\gamma/\omega_0$ determines the length scale of the localization of the field of the unstable mode near the surface of the slab.

An increase in the interaction parameter to $\nu \gg 1$, but under the conditions $\nu^{1/2} \ll \gamma^2$, $(\gamma^2c^2/\omega_p^2 d^2)$, results in a displacement of the maximum of the growth rate up the frequency scale and in a change in its dependence on the beam density. Specifically, under these conditions Eq. (1) reduces to a fourth-degree equation in $(\delta k)^{1/2}$, where $\delta k = k - (\omega/u) \ll (\omega/u)$ is an increment which determines the slow variation of the amplitudes of the perturbations:

$$\delta k^2 + \nu \frac{\omega^2}{\gamma^4 c^2} = \frac{\sqrt{2}\omega^{3/2}c^{1/2}}{\omega_p^2 d}\delta k^{3/2}. \tag{4}$$

The strong effect of the beam-plasma interaction on the transverse wave number $\kappa \simeq (2\delta k(\omega/c))^{1/2}$ has been taken into account here $|\kappa| \gg (\omega/\gamma c)$. Analysis of Eq. (4) reveals that an instability occurs ($\mathrm{Im}k < 0$) in the frequency band $0 < \omega < \omega_{cr} \ll \omega_p$, where

$$\omega_{\mathrm{cr}} = (2\sqrt{2}/3^{3/4}) \frac{\nu^{1/4}}{\gamma} \frac{\omega_p d}{c} \gg \omega_0 \tag{5}$$

and all four roots of Eq. (4) are complex. Of these roots, only one satisfies the condition that the electromagnetic field be bounded as $x \to \pm\infty$ ($\mathrm{Re}\kappa > 0$) and corresponds to the existence of a transverse flux of electromagnetic energy out of the slab into vacuum: $\mathrm{Im}\kappa < 0$. Using the general solution of Eq. (4), found by the Descartes-Euler method, we find the maximum instability growth rate,

$$|\mathrm{Im}k|_{max} \approx 1{,}05 \frac{\omega_p^2 d}{c^2\gamma^3}\nu^{3/4} \propto n_b^{3/4}, \tag{6}$$

and the corresponding frequency $\omega_m \simeq 0.78\omega_{\mathrm{cr}}$. Here

$$\kappa(\omega_m) \approx (1{,}59 - 0{,}64i)\frac{\omega_p^2 d}{c^2}\frac{\nu^{1/2}}{\gamma^2}. \tag{7}$$

It follows from the last expression that as the beam density is increased, there is a decrease in the length scale of the field localization near the surface of the slab[2]) $(\mathrm{Re}\kappa)^{-1} \propto 1/n_b^{1/2}$). Furthermore, we find the following expression for the ratio S_x/S_z (of components of the Poynting vector), which is a measure of the fraction of the electromagnetic energy which is radiated from the lateral boundaries of the system:

$$S_x/S_z = \frac{c|\mathrm{Im}\kappa(\omega_m)|}{\omega_m} = 0.66\frac{\nu^{1/4}}{\gamma} \gg 1/\gamma. \tag{8}$$

This result is considerably larger than the corresponding ratio in the case $\nu^{1/3} \ll 1$, in which we have $S_{\hat{x}}/S_{\hat{z}} \sim \nu^{1/3}/\gamma \ll 1/\gamma$. This beam-plasma instability could accordingly be called "radiative." Note also that the parameter region under consideration here, (2), (3), corresponds to the absence of an instability in a beam-plasma system with a nonradiating lateral boundary, covered by metal walls.[2,3]

1)This plasma mode also exists in the limit of an infinitely thin slab with a surface plasma-electron density $\sigma_p = \lim(n_p d)$, $d \to 0$.

2)A corresponding effect occurs during the radiation of bounded streams of electron oscillators in vacuum.[7]

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