Induced-spin confinement and high-temperature superconductivity

S. Yu. Khlebnikov
Institute of Nuclear Research, Academy of Sciences of the USSR

(Submitted 12 May 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **49**, No. 11, 628–629 (10 June 1989)

A condition for the quantization of induced spin in $(2+1)$-dimensional systems is derived. This condition is analogous to the Dirac condition for the quantization of magnetic charge. Particles whose induced spin is neither an integer nor a half-integer exist only in bound states. The superconducting phase differs from a spin liquid, which is presently being discussed in the context of high-temperature superconductivity.

The mechanism for high-temperature superconductivity in quasi-2D systems is not known. In strongly correlated electron models[1–3] it is possibly associated with a confinement of holes.[4] In 2D, excitations may acquire some exotic characteristics, e.g., an induced spin which is not automatically quantized as a result of interactions.[5] Laughlin[6] treats hole confinement as a consequence of fractional statistics. Kogan[7] has pointed out a transmutation of spin under the formation of a bound state of fermions in $(2+1)$ QED.

In the present letter we derive a quantization condition according to which an induced spin of propagating excitations can have only an integer or half-integer value. By analogy with the Dirac condition for the quantization of magnetic charge, our condition follows from the single-valuedness of the quantum partition function of the excitation and is therefore actually model-independent. Particles whose spin does not satisfy the condition can exist only in bound states, since the total spin of the system has been quantized. In the case of a normal relationship between the spin and statistics, this conclusion agrees with Laughlin's conclusions.[6] In the case at hand, however, the relationship between the spin and the statistics may be disrupted (more on this below). Furthermore, in an effective topological theory we find that high-temperature superconductivity (confinement) and a spin liquid (an RVB state)[1,2,8] are not a common phase[6] but instead distinct[4] phases.

We consider a point charge of magnitude q which is interacting in $(2+1)$ dimensions with an Abelian gauge field A_μ, which is described by a purely topological action $\theta/16\pi^2\int\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda d^3x$. It can be shown that the field created by a charge at rest has an angular momentum which is directed perpendicular to the plane of the system and which has a magnitude

$$S_{ind}=q^2\ \frac{\pi}{\theta} \tag{1}$$

(an induced spin). In determining the quantum partition function of a point charge, we average the contribution of each closed orbit over all possible configurations of the field A_μ. For the topological action this averaging yields the following quantity in the Euclidean formulation:

$$W=\exp(i\,\frac{4\pi^2}{\theta}\,q^2\phi),\quad \phi=\frac{1}{4\pi}\oint_P\oint_P dX_\mu dY_\nu \epsilon^{\mu\nu\lambda}\frac{(X-Y)_\lambda}{|X-Y|^3}, \tag{2}$$

where P is an arbitrary closed contour in 3D Euclidean space.[9] The quantity ϕ in (2) is the "number of self-linkages" of contour P; it depends on the regularization and can actually have any prespecified value.[10] In our interpretation, this situation corresponds to a situation in which the statistics of the particles in $(2+1)$ dimensions can be arbitrary.[5] Specifically, the phase in (2) may be thought of as a statistical phase which is acquired by two particles when they trade places (it is sufficient to represent the contour P as the boundary of a Möbius strip). Even for a given reqularization method, however, the number of self-linkages is determined only modulo unity.[10] In general, this situation leads to a multivaluedness in the determination of W: $W\rightarrow\exp(2\pi i 2\pi q^2/\theta)W$. If we require that the partition function be independent of the arbitrariness in the count of self-linkages, we find that $2\pi q^2/\theta=2S_{ind}$ is an integer (a quantization condition).

With $q=1$ and $\theta=2\pi$, expression (1) reproduces the result of Ref. 9: $S_{ind}=1/2$. It has been hypothesized[3,4] that a topological theory with parameters of this sort describes excitations of an RVB state in the long-wavelength limit.[8] The fictitious field A_μ incorporates the frustration of the initial spin system.[11] The value $\theta=2\pi$ for RVB is supported by the analogy with the 1/2 quantum Hall effect[12] (the fractional $1/m$ Hall effect is described by a topological action with $\theta=4\pi/m$; Ref. 13, for example). The value $S_{ind}=1/2$ corresponds to a "spinon"[1,2]; according to our criterion, confinement does not occur here.

Confinement does occur with $q=1$ and $\theta=4\pi$: $S_{ind}=1/4$ does not satisfy the quantization condition. Charges can propagate only in pairs, and, according to (1), the intrinsic moment of the pair is unity (a p state; cf. Ref. 7). In an idealized situation, the charges in the pair would have to be at the same point, but this condition is removed when the short-range repulsion is taken into account. The long-range repulsion can destroy the effect. The value $\theta=4\pi$ corresponds to a quantum paramagnet in the terminology of Ref. 4. Our confinement mechanism, however, differs from that proposed there, as well as we can judge.

In summary, a new mechanism has been found for the formation of hole pairs in quasi-2D systems. Superconductivity (confinement) and a spin liquid in the terms of the effective theory for excitations correspond to different values of the parameter, i.e., they are different phases. It is possible that as the original system turns metallic, a phase transition occurs. In this transition the coefficient of the topological term increases to the necessary value as a result of hole loops (cf. Ref. 11). It would be interesting to derive the effective action for some specific elementary models. This problem is presently being analyzed; the results will be published separately.

I wish to thank V. A. Kuz'min, V. A. Matveev, V. A. Rubakov, and M. E. Shaposhnikov for a discussion of various questions pertaining to this paper.

[1]P. W. Anderson, Science **235**, 1196 (1987); P. W. Anderson *et al.* Phys. Rev. Lett. **58**, 2790 (1987).
[2]S. A. Kivelson *et al.*, Phys. Rev. B **35**, 8865 (1987).
[3]I. Dzyaloshinskiĭ *et al.*, Phys. Lett. A **127**, 112 (1988).
[4]P. B. Wiegmann, Phys. Rev. Lett. **60**, 821 (1988).
[5]F. Wilczek, Phys. Rev. Lett. **48**, 1144 (1982); Phys. Rev. Lett. **49**, 957 (1988); F. Wilczek and A. Zee, Phys. Rev. Lett. **51**, 2250 (1983).
[6]R. B. Laughlin, Phys. Rev. Lett. **60**, 2677 (1988); Science **242**, 525 (1988).
[7]Ya. I. Kogan, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 194 (1989) [JETP Lett. **49**, 225 (1989)].
[8]P. W. Anderson, Mater. Res. Bull. **8**, 153 (1973).
[9]A. M. Polyakov, Mod. Phys. Lett. A **3**, 325 (1988).
[10]E. Whitten, Preprint IASSNS-HEP-88/33, Princeton, 1988.
[11]I. E. Dzyaloshinskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 650 (1988) [JETP Lett. **47**, 750 (1988)].
[12]V. Kalmeyer and R. Laughlin, Phys. Rev. Lett. **59**, 2095 (1987).
[13]S. Girvin and A. McDonald, Phys. Rev. Lett. **58**, 1252 (1987).

Translated by Dave Parsons