

# Effect of magnetic field on the dynamics of impurities in metallic systems

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A magnetic field can substantially alter the probability for the hop of an impurity or for a transition in a two-level system in a disordered metal. The effect stems from a change in the energy of the interaction of an impurity atom with conduction electrons when a magnetic field is imposed. A similar dependence of the hopping probability on the superfluid current is predicted in disordered superconductors.

1. The conductance of a small metal sample is extremely sensitive to slight changes in the random potential which scatters electrons, because of a quantum interference.<sup>1,2</sup> This anomalous sensitivity makes it possible to experimentally observe a slow relaxation of individual two-level systems and a diffusion of impurities at low temperatures.<sup>3,4</sup>

In the present letter we examine the inverse effect: the effect of changes in the electron wave functions on the behavior of a specific impurity. This effect also turns out to be anomalously strong. Its physical origin lies in Friedel oscillations of the electron density.<sup>5</sup> In a disordered metal, the electron density is determined by an interference of the waves scattered by different impurities and is therefore a random function of the coordinates. An external magnetic field alters the conditions for this interference and thereby alters the electron density at the given point and thus the energy of an impurity at this point. These changes differ substantially at different points, even if they are close together, so the energy required for the activation of an impurity and thus the probability for its hopping also change.

2. Let us examine the changes in the thermodynamic potential of the sample,  $\Omega\{u(\mathbf{r})\}$

$$\delta\Omega = \Omega\{u(\mathbf{r}) + \delta u(\mathbf{r})\} - \Omega\{u(\mathbf{r})\},$$

which are caused by the hopping of an impurity or by a transition in a two-level system, which has the consequence that the random potential  $u(\mathbf{r})$  at point  $\mathbf{r}$  changes by  $\delta u(\mathbf{r})$ . According to the discussion above,  $\delta\Omega$  is a random function of the magnetic field  $H$ , with a mean value of zero:  $\langle\delta\Omega(H)\rangle = 0$  ( $\langle\dots\rangle$  means an average over realizations of the random potential). To determine the fluctuation amplitude  $\delta\Omega$  as a function of  $H$ , we consider the correlation function

$$K(H, H') \equiv \langle[\delta\Omega(H) - \delta\Omega(H')]^2\rangle = \langle[\delta\Omega(H)]^2\rangle + \langle[\delta\Omega(H')]^2\rangle - 2\langle\delta\Omega(H)\delta\Omega(H')\rangle. \quad (1)$$

In the region of weak localization ( $p_F l = 2\varepsilon_F \tau \gg \hbar$ , where  $l$  and  $\tau$  are the mean free path and mean free time, and  $\varepsilon_F$  and  $p_F$  are the Fermi energy and Fermi momentum of the electrons), the quantity  $\langle\delta\Omega(H)\delta\Omega(H')\rangle$  is given by the diagrams in Fig. 1 in the standard diagram technique. This average is determined by distances which are small in comparison with  $l$  and by electron states which are far from the Fermi level. For fields which are not too strong, on the other hand, the quantity  $K(H, H')$  is determined by the region of large distances and low energies.

We consider a sample with dimensions  $L_x \times L_y \times L_z$  in a magnetic field  $H \parallel z$ . Under the conditions  $L_x, L_y \gg \min\{L_T, L_H\} \gg l$ , where  $L_H = \sqrt{\hbar c/e}$  and  $L_T = \sqrt{\hbar D/2\pi T}$  ( $D = l^2/3\tau$  is the electron diffusion coefficient), we find from Fig. 1

$$K(H, 0) = \frac{4A}{\pi^2} \int \frac{\omega d\omega}{C\tau L_z} \operatorname{cth}\left(\frac{\omega}{2T}\right) \sum_{n=0}^{\infty} \sum_{n_z=-\infty}^{\infty} \{4P_n(E_H) - P_n(2E_H) - 3P_n(E \rightarrow 0)\}; \quad (2)$$

$$P_n(E) \equiv E \operatorname{Re}[-i\omega + D(\pi n_z/L_z)^2 + E(n + \frac{1}{2})]^{-1}; \quad E_H \equiv 2DeH/\hbar c, \quad (3)$$

where  $C$  is the impurity concentration. The numerical coefficient  $A$  is determined by the value of  $\delta u(\mathbf{r})$ . If the impurity is displaced a distance greater than the electron wavelength and also greater than the range of the potential,  $A$  is close to unity. Under the condition  $E_H \ll T$ , expression (2) gives us

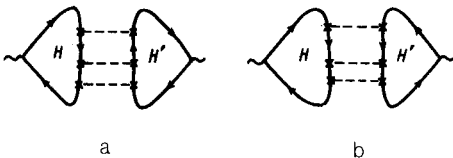


FIG. 1. Feynman diagrams for calculating the correlation function  $\langle\delta\Omega(H)\delta\Omega(H')\rangle$ . Solid lines—Electron Green's functions; dashed lines—average over realizations of the random potential.<sup>6</sup> A vertex corresponds to a change  $\delta u(\mathbf{r})$  in the impurity potential.

$$K(H, 0) = \frac{A}{n_0} \frac{E_H^4}{(2\pi T)^2} \times \begin{cases} \frac{7\zeta(3)}{40\pi^3} & (L_z \gg L_T) \\ \frac{63}{160} (2\pi)^{-3} \zeta\left(\frac{5}{2}\right) & (L_z \ll L_T). \end{cases} \quad (4)$$

Here  $n_0 \equiv Cl^2 \min\{L_T, L_z\}$  is the number of impurities whose displacements change the conductance of the sample, whose dimensions are  $L_T \times L_T \times \min\{L_T, L_z\}$ , by  $e^2/\hbar$  (Refs. 1 and 2), and  $\zeta(t)$  is the Riemann zeta function. If  $E_H \gg T$ , the correlation function  $K(H, 0)$  differs from (4) only by a numerical factor and by the replacement of  $T$  by  $E_H$ :

$$K(H, 0) \propto \frac{AE_H^2}{Cl^2 \min\{L_z, L_H\}} \approx \omega_c^2 \frac{l}{\min\{L_H, L_z\}}, \quad (5)$$

where  $\omega_c = eH\hbar/mc$  is the cyclotron frequency.

These results can be understood by recalling the mesoscopic fluctuations of the state density<sup>7</sup> which lead to fluctuations in the orbital magnetic susceptibility. As a result, the magnetic energy in an ensemble of samples of size  $L$  fluctuates by an amount on the order of  $E_H^2 L^2 / \hbar D$  (Ref. 7). The energy changes by this amount in the case of the displacement of  $n_0 = Cl^2 L$  impurities. The displacement of a single impurity thus alters the magnetic energy by  $E_H^2 L^2 / \hbar D \sqrt{n_0}$ . If we set  $L = \min\{L_H, L_T\}$ , we then find relations (4) and (5).

These relations are valid under the conditions  $T, E_H \lesssim \hbar/\tau$ , so the maximum change in the impurity activation energy in a magnetic field, on the order of

$$|\delta\Omega(H) - \delta\Omega(0)|_{max} \approx \sqrt{K(H, 0)}_{max} \approx (Cl^3)^{-1/2} \frac{\hbar}{\tau} \approx \frac{\hbar^2}{\tau^2 \epsilon_F}, \quad (6)$$

can substantially exceed the temperature  $T$ . The relaxation time of the given two-level system changes by a large factor in a magnetic field.

When a magnetic field is applied, the suppression of the Cooper component (Fig. 1b) reduces the dispersion of the electron component of the impurity energy,

$$\langle \delta\Omega(0)^2 \rangle - \langle \delta\Omega(H)^2 \rangle \approx K(H, 0),$$

and thus the total scatter in the impurity energy  $E_a$  at the various positions of the impurity and the crystal. The effective activation energy for impurity diffusion,  $E_a$ , thus decreases by  $\delta E_a \approx K(H, 0)/E_a$  (Ref. 8). Although this quantity is smaller than  $|\delta\Omega(H) - \delta\Omega(0)| \approx \sqrt{K(H, 0)}$ , it may be comparable to  $T$ . The impurity diffusion coefficient depends strongly on the magnetic field.

3. If the electron system is in a superconducting state, the impurity energy depends in a random way on the superfluid current.<sup>1)</sup> As a current  $J_s$  on the order of the

critical current  $J_c$  flows through the superconductor, the impurity activation energy changes, in the case  $\Delta > T$ , by

$$|\delta\Omega(J_s) - \delta\Omega(0)| \approx \Delta(Cl^2 \min\{\xi, L_z\})^{-1/2} = \Delta n_0^{-1/2}, \quad (7)$$

where  $\Delta$  is the superconducting gap, and  $\xi = \sqrt{D\hbar/\Delta}$  is the coherence length.

This effect is analogous to the magnetic-field dependence of  $\delta\Omega$ , which was discussed above: The flow of a current on the order of  $J_c$  has the consequence that the phase acquired by an electron over the coherence length changes by an amount on the order of  $\pi$ .

There is another way to explain (7): At a given current  $J_s$ , the energy of the superconductor depends on  $N_s$ , the density of superconducting electrons ( $\Omega = mJ_s^2\xi^3/N_s e^2$ ). The hopping of an impurity alters the superconducting density in a volume of  $\xi^3$ . The relative change in  $N_s$  and thus in the energy is given in order of magnitude by<sup>9</sup>

$$\frac{\delta\Omega}{\langle\Omega\rangle} \approx \frac{\delta N_s}{\langle N_s\rangle} \approx \frac{\delta G_\xi}{\langle G_\xi\rangle} \approx \frac{e^2}{\hbar} (\langle\Omega\rangle \sqrt{n_0})^{-1}, \quad (8)$$

where  $G_\xi$  is the conductance of a cube of side  $\xi$  in the normal state.

4. The entire discussion above applies not only to impurity atoms but also to the ions which make up the crystal lattice of a metal. In a magnetic field, each ion is displaced in a random direction by an amount on the order of

$$\delta x \approx \sqrt{K(H, 0)} (\omega_D^2 M a)^{-1} \approx \frac{a\omega_c}{M} (a\omega_D)^{-2} \sqrt{\frac{l}{\min\{L_H, L_z\}}} \approx a \sqrt{\frac{l}{\min\{L_H, L_z\}}} \frac{\omega_c}{\epsilon_F},$$

where  $a$  is the lattice constant,  $M$  is the mass of the ion, and  $\omega_D$  is the Debye frequency.

Because of this effect, the structure factor of the lattice becomes dependent on the magnetic field or on the superfluid velocity in the superconductor. In principle, this dependence could be studied by x-ray diffraction or by a neutron method. An alternating magnetic field results in the excitation of phonons at the doubled frequency [because of the relation  $\delta\Omega(\mathbf{H}) = \delta\Omega(-\mathbf{H})$ ].

5. We note in conclusion that this entire analysis has been carried out in the adiabatic approximation. We believe that incorporating nonadiabatic effects<sup>11</sup> could not lead to an exponential dependence on the temperature; it could only alter the coefficient of the exponential function. For this reason, the change in the activation energy which we have been discussing here is the dominant change, at least at low temperatures.

<sup>11</sup>This dependence was discussed by Kozub<sup>10</sup> for classical Josephson junctions.

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