Vortex dynamics in high-T_c superconductors near the upper critical field

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The critical pinning current of a homogeneous layered superconductor has been calculated for the case in which the vortices are moving across the layers. The current-voltage characteristic has been determined for a weakly coupled layered superconductor.

1. The resistive properties of high-T_c superconductors in a magnetic field are of considerable interest in view of the difficulty of measuring the critical magnetic fields. An important factor in this case is the vortex pinning. Pinning usually occurs because of the inhomogeneity of a sample. Most of the high-T_c superconductors are strongly anisotropic, and some of them, such as BiSrCaCuO, display properties of layered compounds.[1] Layered superconductors are expected to have a finite pinning even in the case of homogeneous samples if the Lorentz force is perpendicular to the layers.

In the present study we have calculated the critical pinning current for layered superconductors near H_{c2} when the magnetic field is oriented in the basal *ab* plane of the crystal (in the plane of the layers). The current flows along the layers at right angles to **H**. We have studied a weakly coupled layered superconductor ($\xi_c \gg s$) and a strongly coupled layered superconductor ($\xi_c \sim s$), where ξ_c is the coherence length along the *c* axis, and *s* is the spacing between the layers. We have also calculated the current-voltage characteristic of a weakly coupled layered superconductor. The effects attributable to the flux creep[2] were ignored. The calculations were based on the Ginzburg-Landau theory for layered superconductors.[3,4] The time-dependent Ginzburg-Landau equations were used to calculate the *V-I* characteristic. The choice of such a

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model is justifiable by the fact that high-T_c superconductors appear to have a fairly broad temperature region near T_c, where there is a gapless superconductivity.

Le us assume that the z axis runs along the c axis of the crystal, that the magnetic field is directed along the y axis, and that the transport current flows along the x axis. Near H_{c2} we set $\varphi = -Ex$; $\mathbf{A} = (0; 0; -Hx)$. Expanding the order parameter of the nth layer in a Fourier series

$$\psi_n = \sum_q e^{iqsn}\psi_q(x, t),$$

we find for $\psi_q(x, t)$ (cf. Ref. 4)

$$u\left(\frac{\partial}{\partial t} - 2ieEx\right)\psi_q + \hat{H}_0\psi_q + \beta\sum_{q_1, q_2}\psi_{q_1}\psi^*_{q_2}\psi_{q-q_1+q_2} = -\alpha\psi_q, \tag{1}$$

where

$$\hat{H}_0 = -\frac{1}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{Ms^2}\left[1-\cos\left(qs + \frac{2eHsx}{c}\right)\right]. \tag{2}$$

Here $\alpha = a(T - T_c)$, $|\alpha|^{-1} = 2m\,\xi_{ab}(T)$, where $\xi_{ab}(T)$ is the coherence length in the ab plane, m is the quasiparticle mass in the ab plane, M is the effective Ginzburg-Landau mass along the c axis: $\xi_c/\xi_{ab} = (m/M)^{1/2} \ll 1$, and u is the "viscosity coefficient." In the dirty limit we have $\xi^2_{ab} = \pi D/8(T_c - T)$ and $u = (2mD)^{-1}$, where $D = v_F^2\,\tau/2$ is a two-dimensional diffusion coefficient. From the quantities in (2) we can construct the parameter[4] $h = (2eHs^2/c)(M/m)^{1/2}$. The weakly coupled layered superconductor, $\xi_c(T) \gg s$, corresponds to $h \ll 1$ and the case $h \gg 1$ corresponds to a strongly coupled layered superconductor, $\xi_c(T) \approx s/\sqrt{2}$.

2. Let us consider a weakly coupled layered superconductor with $h \ll 1$, having in mind compounds of the type YBaCuO (Ref. 5). The operator (2) in this case has narrow energy bands. We expand ψ_q in Wannier functions (k is the band number and l is an integer) of the operator H_0:

$$\psi_q(x, t) = \sum_{k, l} f^{(k)}(l, t)\,\varphi_k\left(x + \frac{qc}{2eH} - \frac{\pi lc}{eHs}\right).$$

The Wannier functions $\varphi_k(k)$ are closely related to the wave functions of a linear oscillator, which corresponds to an expansion of the cosine in (2). The envelope $f^{(0)}(x, t)$ for the lower band is determined from the expression[6]

$$\left[u\frac{\partial}{\partial t} + \epsilon_0(p) - iu(2eEx - v_L q) - |\alpha|\right] f^{(0)}(x, t) = 0,$$

where $p = -i(c/2eHs)(\partial/\partial x)$; we set $x = \pi lc/eHs$. The spectrum $\epsilon_0(p)$ is a narrow band whose center is at the lower oscillator level:

$$\epsilon_0(p) = \frac{h}{2Ms^2} - \frac{\Delta}{2}\cos(2\pi p);\quad \Delta = \frac{16h^{1/2}}{\pi^{1/2}Ms^2}\exp(-8/h).$$

Taking the next band into account involves replacing the argument of the Wannier function $\varphi_0(x)$ with $x - icv_L Mu/2eH$, where $v_L = cE/H$ is the average velocity of the vortices. Finally,

$$\psi_q = C_q \exp[-iv_L qt + (\frac{1}{\xi_c^2} - \frac{h}{s^2})\frac{t}{2Mu} + \frac{\Delta s}{4\pi u v_L}\sin(2\pi p + \frac{2\pi v_L t}{s})]\Psi^{(0)}_{p+v_L t/s}(x + \frac{qc - icv_L Mu}{2eH}), \quad (3)$$

where $\Psi_p^{(0)}(x)$ is the Bloch function of the lower band of the operator $\hat{H}_0$. The quasimomentum p is chosen on condition that there is no solution which depends exponentially on time. At $E=0$ we thus find $\epsilon_0(p) = |\alpha|$, from which we find $H_{c2}(p)$. The quasimomentum p corresponds to the band velocity $v_p = \partial\epsilon_0(p)/\partial p$ and to the superconducting current which has been determined. The maximum value of v_p and of the current was measured at $p = 1/4$. In this case $H_{c2} = c/2e\xi_{ab}\xi_c$. At $E \neq 0$ we must also set $p = 1/4$. Normalization of the coefficients C_q in (3), which satisfy the periodicity condition, is found, as usual, from nonlinear equation (1). Calculation of the space- and time-averaged current flowing along the x axis gives

$$j_x = j_c \frac{I_1(E_0/E)}{I_0(E_0/E)} + \sigma_f E; \quad \sigma_f = \sigma(1 + \frac{\pi^4}{28\zeta(3)} \circ \frac{H_{c2} - H}{\beta_L H_{c2}}). \quad (4)$$

Here $I_{0,1}$ are the modified Bessel functions,

$$E_0 = \frac{4\exp(-8/h)}{\pi^{3/2} mu\xi_{ab}^3 e}, \quad j_c = \frac{\Delta mc^2}{8se\kappa^2\beta_L}(m/M)^{1/2}(1 - \frac{H}{H_{c2}});$$

$\kappa = \lambda_{ab}/\xi_{ab}$, and β_L is the lattice constant. In the case of a triangular lattice we have $\beta_L = 1.16$. The expression for the conductivity σ_f with a current flow (4) is the well-known result for a gapless superconductor (see, e.g., Ref. 7). The maximum dissipationless current j_c that can flow without the motion of vortices is the critical pinning current. This current is proportional to the band width Δ and vanishes in the absence of layers, $h \to 0$. The initial part of the I-V characteristic, $E \ll E_0$,

$$j_x = j_c + \sigma(1 - \frac{\pi^6}{28\zeta(3)} \times \frac{H_{c2} - H}{h\beta_L H_{c2}})E$$

may have an H-dependent negative slope, giving rise to a depinning and to the appearance of hysteresis on the I-V curves. At $E \gg E_0$ the I-V characteristic begins to behave normally, $j = \sigma_f E$.

3. Let us now consider a strongly coupled layered superconductor, $h \gg 1$ and $\xi_c(T) \approx s/\sqrt{2}$. At $E=0$ the equation for ψ_q reduces to $\hat{H}_0\psi_q = |\alpha|\psi_q$, where the "potential energy" in H_0 is small. We find

$$\psi_q = C_q \exp(i\frac{2eHspx}{c})\, w(x + \frac{qc}{2eH}),$$

where $w(x)$ is the Mathieu function, and the eigenvalue for $h \gg 1$ and $ph^2 \ll 1$ is $\epsilon(p) = (1 - h^{-2} + p^2 h^2/2)/Ms^2$. From the condition $\epsilon(p) = |\alpha|$ we find

$$H_{c2}(p)/H_{c2} = 1 - p^2 h^4/4,$$

where[4]

$$H_{c2} = \frac{c}{2es^2}(m/M)^{1/2}\left[1 - \frac{s^2}{2\xi_c^2(T)}\right]^{-1/2}.$$

The space-averaged current along the x axis, which corresponds to the quasimomentum p, is

$$j_x(p) = (m/M)^{3/2}\,\frac{c^2 hp}{2\pi e\kappa^2 s^3 \beta_L} \times \frac{H_{c2}(p) - H}{H_{c2}}. \tag{5}$$

The maximum value of the current (5), expressed as a function of p, is the critical pinning current,

$$j_c = \frac{2c^2}{3\sqrt{3}\pi e s^3 \kappa^2 h \beta_L}(m/M)^{3/2}\left(1 - \frac{H}{H_{c2}}\right)^{3/2}.$$

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