

Frequency dispersion of the conductivity of metal-oxide superconductors

I. O. Kulik

Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukrainian SSR

(Submitted 25 April 1989; resubmitted 29 May 1989)

Pis'ma Zh. Eksp. Teor. Fiz. **49**, No. 12, 683–685 (25 June 1989)

The conductivity of local pairs in the normal state of a metal-oxide superconductor is inversely proportional to the temperature T and decreases with the frequency in accordance with $\sigma(\omega) = \sigma_0 / (1 + \omega^2/\omega_0^2)$, where the cutoff frequency is $\omega_0 = \Gamma_0 T/T_c$ and $\Gamma_0 \sim 10^{11} \text{ s}^{-1}$. The observation of this dependence at low frequencies would be evidence of an exotic (electron-pair) mechanism for the mobility of metal-oxide compounds and of a Bose condensation of pairs as a possible mechanism for high-temperature superconductivity.

A mechanism for high-temperature superconductivity which is presently under discussion is similar to the Shafroth-Butler-Blatt mechanism¹ but differs from it in having two subsystems: band carriers and local electron (or hole) pairs, which interact strongly with each other and which cause an instability in the channel of Bose condensation of pairs. This mechanism leads to the prediction of a resistance component which is linear in the temperature above the superconducting transition temperature.^{2,3} This component may be comparable in magnitude to (or even greater than) the component from band carriers, since the latter have a short relaxation time $\tau_1 \sim 10^{-14} \text{ s}$ as a result of the pronounced scattering by local pairs and lattice defects, and they are close to the threshold for Anderson localization.⁴ The local pairs remain tightly bound at all temperatures, including temperatures above T_c . Their decay time, i.e., the time over which they convert into two electron (or hole) excitations in the conduction band, is $\tau_2 \sim 10^{-10} - 10^{-11} \text{ s}^{-1}$, substantially longer than the relaxation time of one-electron states. As a result, it turns out that despite the small width (W_2) of the effective electron-pair band, the product $W_2\tau_2 \sim \epsilon_F/T$ is large in comparison with unity (the width of the one-electron band is $W_1 \sim \epsilon_F$, and we have a product $W_1\tau_1 \lesssim 1$). The pairs are not canonical quasiparticles (Fermi or Bose quasiparticles) and do not obey an ordinary kinetic equation. Nevertheless, and in contrast with the hopping conductivity in a semiconductor, the pair conductivity is not an exponential function of the temperature, and it has a strong frequency dependence at $\omega\tau_2 \sim 1$. A localization of the pairs is possible only as a result of a diagonal disorder (fluctuations of the pair energy in space).

We link the band excitations in metal-oxide compounds of the $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ type with holes at oxygen in copper-oxygen planes, while we link the local pairs with peroxide complexes O_2^{2-} or with neutral oxygen atoms (i.e., a pair of holes in a filled p shell of an O^{2-} ion) positioned between planes. The hybridization of local states with band states leads to a joint Cooper condensation in the band and to a Bose condensation in the electron-pair system. The critical transition temperature [we are omitting

the slow logarithm $\ln(\epsilon_0/T)$ from the expression for T_c from Refs. 2 and 3] is given by

$$T_c \approx N(\epsilon_F) \langle |V|^2 \rangle \frac{1 - 2\nu}{\ln[(1 - \nu)/\nu]}, \quad (1)$$

where V is the hybridization constant, and ν is the average number of pairs at a site, which is determined by the doping level x .

Examining the kinetic equation for the pair density matrix $\rho_{ij} = \langle A_i^+ A_j \rangle$,

$$\partial \rho_{ij} / \partial t + 2ieE(R_i - R_j) \rho_{ij} = \hat{I}_{ij}, \quad (2)$$

$$\hat{I}_{ij} = - \int_{-\infty}^0 dt \langle [H_{int}(t), [H_{int}, A_i^+ A_j]] \rangle,$$

where H_{int} is the hybridization Hamiltonian, and $A_j^+ (A_j)$ is the operator which creates (annihilates) a pair at a site (at an atomic cluster) with coordinate R_j , we find the relaxation time

$$\tau_2^{-1} \approx \Gamma_0 T / T_c, \quad \Gamma_0 = 2\pi N(\epsilon_F) W_2^2 \quad (3)$$

and an estimate of the effective mass of the pair (m_1 is the band mass, i.e., the one-electron mass):

$$m_1 / m_2 \approx N(\epsilon_F) W_2, \quad W_2 = N(\epsilon_F) V^2, \quad (4)$$

where $N(\epsilon_F)$ is the state density of holes at the Fermi level.

From (2) we find an expression for the current J_{ij} which flows from site i to site j :

$$J_{ij} = ie(\gamma_{ij}^* \rho_{ij} - \gamma_{ij} \rho_{ji}), \quad (5)$$

$$\gamma_{ij} = N^2(\epsilon_F) V^2 \int d\xi_k \int d\xi_{k'} \frac{1 - n_k - n_{k'}}{\epsilon_k + \epsilon_{k'} - 2E_0 + i\delta} e^{i(k - k')(R_i - R_j)},$$

where

$$n_k = (e^{\beta(\epsilon_k - \mu)} + 1)^{-1}, \quad \mu = E_0 - \frac{T}{2} \ln \frac{1 - \nu}{\nu},$$

and E_0 is the energy level of a pair (per hole) between the bottom and top of the band. The pair conductivity of the frequency ω is

$$\sigma_2(\omega) = \sigma_2^0 / (1 - i\omega\tau_2), \quad (6)$$

where

$$\sigma_2^0 \approx \frac{ne^2 \hbar}{m_1 T}, \quad n = n_0 \nu. \quad (7)$$

This result can be understood in the spirit of the Drude formula for the conduc-

tivity $\sigma_2^0 = ne^2\tau_2/m_2$. Since τ_2 and m_2 depend on the interaction constant in the same way according to (3) and (4), this constant cancels out, and the expression for the conductivity takes the simple form described by (7) (without the logarithm). A more accurate expression for the resistance is

$$\rho = \rho_0 \frac{T}{T_c} + \rho', \quad \rho_0 = m_1 W_2 / ne^2 \hbar, \quad (8)$$

where ρ' is the "residual" resistance due to scattering by the disorder.

Adopting $m_1 = 5m_0$, $N^{-1}(\epsilon_F) \sim \epsilon_F \sim 10^4$ K, and $V \sim 10^3$ K, which corresponds to $T_c \sim 10^2$ K according to (1), to find a numerical estimate, we find $\rho_0 \sim 10^{-4} \Omega \cdot \text{cm}$ and $\Gamma_0 \sim 1$ K. In other words, we find the "cutoff frequency" of the pair mobility to be $\omega_0 \approx \Gamma_0 / \hbar \sim 10^{11} \text{ s}^{-1}$. In the same approximation, the effective mass of the pair is $m_2 \sim 10^2 m_1$, and the mean free path is $l_2 \sim \hbar v_F / T \sim 10^{-5} \text{ cm}$ at $T \approx T_c$. These estimates of course refer to values averaged over directions. They do not rule out a higher mobility (and lower values of m_2) in certain individual directions, e.g., along cleavage plane in layered crystals.

We can thus draw the following conclusions regarding the electron-pair component of the conductivity of a metal-oxide superconductors: a) It is extremely large in absolute value and is capable by itself of explaining the correct order of magnitude of the resistance. b) It is inversely proportional to the temperature, regardless of the relation between the width of the electron-pair "band" and the temperature. c) It exhibits a substantial frequency dispersion at frequencies considerably lower (by several orders of magnitude) than the dispersion frequency of the one-electron mobility. The latter circumstance means that the electron-pair component of the conductivity can be distinguished from the ordinary (band) mobility and that the presence of local pairs in high-temperature superconductors can be established.

Note in this connection that a decrease in the conductivity in the normal state of the compounds $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ and $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$, beginning at frequencies $\omega \sim 10^{10} \text{ s}^{-1}$, was recently observed.^{6,7} The subsequent growth of σ at higher frequencies ($\omega > 10^{11} \text{ s}^{-1}$), however, is surprising and does not fit into the picture which is being drawn. Nevertheless, the very fact that there is a dispersion of the conductivity of metal oxide compounds at such low frequencies indicates that the one-electron approximation is not valid as a model for a high-temperature superconductor and that we need to incorporate a "slow" (inert) electron system in order to describe its metallic properties.

¹M. Shafroth *et al.*, *Helv. Phys. Acta* **30**, 93 (1957).

²I. O. Kulik, *Fiz. Nizk. Temp.* **13**, 879 (1987) [*Sov. J. Low Temp. Phys.* **13**, 505 (1987)]; **14**, 209 (1988) [*Sov. J. Low Temp. Phys.* **14**, 116 (1988)].

³I. O. Kulik, *Int. J. Mod. Phys. B* **1**, 851 (1988).

⁴V. V. Moshchalkov *et al.*, *Fiz. Nizk. Temp.* **14**, 988 (1988) [*Sov. J. Low Temp. Phys.* **14**, 543 (1988)].

⁵L. F. Mattheiss and D. R. Hamann, *Solid State Commun.* **63**, 395 (1987).

⁶A. S. Shcherbakov *et al.*, *Pis'ma Zh. Eksp. Teor. Fiz.* **46**, 111 (1987) [*JETP Lett.* **46**, 136 (1987)]; *Fiz. Met. Metalloved.* **64**, 735 (1987).

⁷A. S. Shcherbakov *et al.*, *Pis'ma Zh. Eksp. Teor. Fiz.* **49**, 102 (1989) [*JETP Lett.* **49**, 121 (1989)].