Nonzero state density in superconductors with a high transition temperature

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High-T_c superconductors can have a Fermi surface even in their superconducting state. This result explains the observed term in the heat capacity of high-temperature superconductors which is linear in the temperature. This term indicates a nonzero state density.

Measurements of the heat capacity of the high-temperature supeconductor $YBa_2Cu_3O_7$ reveal a linear function of the temperature, $C = \gamma T$, at low temperatures, with a value $\gamma = 4.5$ mJ/(mol·K^2) for the purest samples.[1,2] It is becoming more and more obvious that a linear term and thus a nonzero state density are characteristic of the superconducting state in this substance.[3] This circumstance may be taken as an argument in favor of a new type of superconductivity, based on the model of resonating valence bonds (RVB).[4] On the other hand, one could suggest an alternative explanation: that a nonzero state density is a typical property of even ordinary superconductivity, provided that the band-band interaction is sufficiently strong.

In the case of interacting bands the gap function depends on the band indices n and $m[\Delta_{mn}(\mathbf{k})]$, and the spectrum is no longer determined by the standard formula $E(\mathbf{k}) = \pm(\epsilon^2(\mathbf{k}) + |\Delta^2(\mathbf{k})|)^{1/2}$, which is valid only in the single-band approximation and according to which the spectrum in the single-band approximation vanishes at no $\mathbf{k}$ in the case of an ordinary superconductivity. The situation changes if the pairing interaction is strong, and functions Δ_{nm} which are not diagonal in the band indices must be taken into account. These functions are usually ignored in the weak-coupling limit, since they are small by a factor proportional to the parameter T_c/E_F.

In general, the quasiparticle spectrum in a superconductor is determined by the eigenvalues of a Bogolyubov matrix of the general form

$$H = \begin{pmatrix} \epsilon_n(\mathbf{k})\delta_{nm} & \Delta_{nm}(\mathbf{k}) \\ \Delta^*_{mn}(\mathbf{k}) & -\epsilon_n(-\mathbf{k})\delta_{nm} \end{pmatrix}, \qquad (1)$$

where $\epsilon_n(\mathbf{k})$ is the electron spectrum of band n. The qualitative behavior of the spectrum is determined by the general properties of a Hermitian matrix, according to which the matrix has topologically stable zero eignevalues $E(\mathbf{k}) = 0$, which do not disappear even when external perturbations are acting. These zeros in the quasiparticle spectrum are analogous to topologically stable defects in condensed media[5] but differ from the latter in that they exist in momentum space (more precisely, in the space of the quasimomentum $\mathbf{k}$) rather than in real space.

The zeros of the spectrum are defects of the matrix H in momentum space, i.e., a subset of momentum space in which the matrix H is degenerate $[\det H(\mathbf{k}) = 0]$. The zeros are described by classes of nonequivalent mappings of momentum space into the space of nondegenerate matrices H, which form homotopic groups π_0, π_1, π_2,... . Group π_0 describes classes of singular surfaces in momentum space on which we have $\det H = 0$, so at least one branch of the spectrum intersects the energy zero. These are thus Fermi surfaces, by definition. They correspond to domain walls in a ferromagnet, since they separate regions in which Bogolyubov quasiparticles have a positive energy (the Bogolyubov spin is up) from regions in which they have a negative energy (the Bogolyubov spin is down). The topological invariant for these domain walls is expressed in terms of a Green's function:

$$G = (i\omega - H)^{-1}; \qquad N(\mathbf{k}) = \mathrm{tr}\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi i}\, G(\mathbf{k},\omega)\partial_\omega G^{-1}(\mathbf{k},\omega). \qquad (2)$$

This integer invariant changes discontinuously when the momentum $\mathbf{k}$ crosses the Fermi surface. In a normal Fermi liquid, $N(\mathbf{k})$ would be equal to the number of quasiparticle states: $N(\mathbf{k}) = 1$ for $\mathbf{k}$ inside the Fermi surface and $N(\mathbf{k}) = 0$ for external momentum. According to Luttinger's theorem,[6] the total number of fermions, $\Sigma_{\mathbf{k}} N(\mathbf{k})$, is equal to the phase volume under the Fermi surface. The discontinuity in $N(\mathbf{k})$ also induces a jump in the momentum distribution of the real particles.

In pair-correlated systems, Luttinger's theorem does not hold, since the generalized Green's function includes an anomalous Gor'kov function F, so $\Sigma_{\mathbf{k}} N(\mathbf{k})$ is not equal to the number of particles. Nevertheless, a Fermi surface can again be defined as a singular surface on which the invariant $N(\mathbf{k})$ has an integer discontinuity, which induces a (noninteger) discontinuity in the particle distribution function. Incorporating electron-electron Fermi-liquid correlations does not alter Eqs. (2); in this case the matrix $G(\mathbf{k},\omega)$ must be treated as a complete one-particle Green's function of the system, which depends on the imaginary frequency and whose indices include the spin, the band index, and the Bogolyubov spin.

In addition to singular surfaces, there may be singular lines and singular points in the fermion spectrum in a superconductor. These singular features would be described by higher-index homotopic groups, π_1 and π_2, respectively. In the case of a matrix H of general form, group π_1 is trivial; i.e., the lines of zeros are topologically unstable.[7] They may exist by virtue of a certain symmetry of the superconducting state, but they disappear in the case of perturbations which break the symmetry. Group π_2 is nontrivial. It provides classes of topologically stable point zeros, which do not disappear even if the symmetry is broken.[8] Such points (boojums) are described by an integer invariant, also in terms of a Green's function:

$$N = \frac{1}{24\pi^2} e^{ijkl}\, \mathrm{tr}\int dS_i G(x)\partial_j G^{-1}(x) G(x)\partial_k G^{-1}(x) G(x)\partial_l G^{-1}(x). \qquad (3)$$

This equation remains in force when electron-electron Fermi-liquid correlations are taken into account.

Singular (Fermi) surfaces are the most typical situation in condensed media, since they exist even in a normal metal. The disappearance of such a surface or a change in its topology is realized by a Lifshitz phase transition. A transition into a superconducting state or into some other state with a long-range order of the spin-density-wave type or the charge-density-wave type is usually also accompanied by a Lifshitz transition. Whether the Fermi surface disappears or simply changes shape or topology upon such a transition depends on the details of the system. The particular example in which a small Fermi surface survives the formation of a spin density wave is discussed by Kato and Machida[9] with respect to heavy-fermion superconductors.

In the superconducting state, the Fermi surface disappears completely in the case of a single band, but it may survive if the band-band hybridization due to the superconductivity $[\Delta_{mn}(\mathbf{k})]$ is sufficiently pronounced, i.e., comparable to the distance between bands. Since this hybridization is small to the extent that T_c/E_F is small, a residual Fermi surface can be expected only in high-temperature superconductors. Consequently, there will be a gapless superconductivity even in the absence of impurities and even at $T=0$.

It is possible that the term which is linear in T in the heat capacity of $YBa_2Cu_3O_7$ reflects a nonzero state density due to the presence of a Fermi surface in the superconductivity state. This interpretation agrees with experimental results on Raman scattering,[10] which indicate the presence of normal electrons at low T. Since the size of the residual Fermi surface is sensitive to the details of the interaction, we would expect the linear term to depend strongly on the pressure, the magnetic field, the deformation of the crystal, and other perturbations. We do not rule out the possibility that it would disappear at certain critical values of the parameters.

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[1]M. E. Reeves *et al.*, Phys. Rev. B **35**, 7207 (1987).
[2]M. Ishikawa *et al.*, Physica C **153-155**, 1089 (1988).
[3]S. E. Stupp and D. M. Ginsberg, *Review of the linear term in the low temperature specific heat of* $YBa_2Cu_3O_{7-\delta}$, Preprint in the University of Illinois at Urbana-Champaign.
[4]P. W. Anderson, Science **235**, 1196 (1988).
[5]N. D. Mermin, Rev. Mod. Phys. **51**, 591 (1979).
[6]J. M. Luttinger and J. C. Ward, Phys. Rev. **118**, 1417 (1960); J. M. Luttinger, Phys. Rev. **150**, 202 (1966).
[7]P. G. Grinevich, J. Low Temp. Phys. **72**, 371 (1988).
[8]G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **46**, 81 (1987) [JETP Lett. **46**, 98(1987)].
[9]M. Kato and K. Machida, Phys. Rev. **37**, 1510 (1988).
[10]S. L. Cooper *et al.*, J. Opt. Soc. Am. B **6**, 436 (1989).

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