# Use of superposition model in analyzing muon groups generated by high-energy nuclei of the primary cosmic rays

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The Monte Carlo method is used to analyze muon groups generated by the nuclei of the primary cosmic rays. A comparison of the characteristics of showers generated by nuclei and protons leads to the conclusion that the superposition model is valid. Calculations have been carried out for $E_\mu = 0.2$ TeV, which corresponds to the lowest threshold energy for the detection of muons at the Baksan underground scintillation telescope.

Several methods available for studying the chemical compositon of the high-energy primary cosmic rays are based on both an analysis of data on the muon component of extensive air showers and a joint analysis of this component and other components of extensive air showers. Most of these methods are based on the superposition model which was originally proposed by Goryunov *et al.*[1] According to this model, an extensive air shower resulting from a primary nucleus of energy $E_A$ and atomic weight $A$ is equivalent to the sum of $A$ showers from primary protons with energies $E_0 = E_A/A$. In the energy region accessible to research on the basis of extensive air showers, only theoretical predictions based on certain models are possible with respect to the superposition model. In modeling showers, for example, we have assumed that $\overline{N}_A$ nucleons of the primary nucleus interact in the first event, while the other $A - \overline{N}_A$ nucleons begin to interact as free nucleons at the height of this first event. Such a model is of course approximate, since it ignores the fluctuations in $\overline{N}_A$ and also the production of secondary nuclei (fragments). It does nevertheless make it possible to take into account the basic factor characteristic of showers generated by nuclei: the $A$ dependence of the height of the first interaction.

All of the characteristics of the elementary event are taken into account in accordance with the model of quark-gluon strings.[3–5] Data found from this model for the $pp$ and $pA$ interactions give a satisfactory description of experimental results at accelerators and in the cosmic rays. This model was recently generalized to nucleus-nucleus interactions by Shabelsky.[6]

In accordance with the results of Ref. 6, the cross section for the inelastic interaction of a primary nucleus $A$ with an air nucleus ($\overline{A}_a = 14.7$) was chosen in the form

$$\sigma_A(E_0) = 273\sqrt{A}(1 + 0.068\ln E_0)\ \text{mb}.$$

If we adopt the multiple-scattering model,[6,7] the average number of interacting nucleons of the incident nucleus is

$$\overline{N}_A = A\sigma^{in}_{NA_a}/\sigma^{in}_{AA_a}. \qquad (1)$$

The value of $\overline{N}_A$ depends strongly on $A$ and weakly on $E_0$ over the range $E_0 = 10$–$10^3$ TeV/nucleon. Ignoring the $E_0$ dependence in this range, we find $\overline{N}_{56} = 8$ and $\overline{N}_4 = 2$. Note also that in this $E_0$ range the inclusive spectra of the secondary hadrons produced by nucleons of the primary nuclei agree highly accurately with the corresponding spectra for primary protons.

In experiments on the muons of extensive air showers, the basic observable characteristics are usually $n_A$, the number of muons, and the spatial distribution of these muons. If we assume that the average transverse momenta of the secondary hadrons are independent of $A$, we conclude that the spatial distributions of muons generated by different nuclei with identical energies per nucleon will be the same in a first approximation if the average heights at which the muons are produced, $\overline{h}_A$, are the same. From this standpoint one might suggest that the superposition model is correct if the average range of one nucleon of the primary nucleus is equal to the average range of a primary proton before an interaction, i.e., if

$$\overline{N}_A\lambda_A/A + (A - \overline{N}_A)(\lambda_A + \lambda_p)/A = \lambda_p.$$

It is easy to show that expression (1) is a solution of this equation for $\overline{N}_A$, so we would expect that the superposition model would hold highly accurately for $\overline{h}_A$ and $\overline{n}_A$.

Calculations have been carried out for primary nuclei with $A = 1$, 4, and 56, with allowance for the production of muons through charged pions and kaons. A detailed description of the Monte Carlo algorithms for the primary protons is given in Ref. 8. Table I shows ratios of the average heights for nuclei, $\overline{h}_A$, to the average height for primary protons, $\overline{h}_1$. Also shown here are ratios of the average number of muons generated by one nucleon of a primary nucleus, $\overline{n}_A/A$, to the value of $\overline{n}_1$ for the primary protons. The statistical errors in the values given in this table are less than 1% (except for the two points where the errors are specified).

A question of obvious interest is that of fluctuations in the muon multiplicity as a

TABLE I.

| $E_0$, TeV/nucleon | 10 | $10^2$ | $10^3$ |
|---|---|---|---|
| $\bar{h}_4/\bar{h}_1$ | 1,00 | 0.99 | 1,00 |
| $\bar{n}_4/4\bar{n}_1$ | 1,00 | 1.00 | 1,00 |
| $\bar{h}_{56}/\bar{h}_1$ | 1,01 | 1.01 | 1,02 |
| $\bar{n}_{56}/56\bar{n}_1$ | 1,02 | 1.02 | 1,02 |
| $D_4/4D_1$ | 1,16 | 1,15 | 1,22 |
| $D_{56}/56D_1$ | 1,64 | 2,26 ± 0,07 | 1,69 ± 0,17 |

function of $A$. Figure 1 shows the relative fluctuations in $n_A$ for various values of $A$ as a function of $E_0$. Under the assumption that the superposition model is valid for the average multiplicity and its fluctuations in accordance with a Poisson law, one can show that a measure of the dispersion for a nucleus $A$ can be expressed in terms of that for a proton by $D_A = AD_1$. Table I shows values of these measures of the dispersion, normalized to the value of $AD_1$, versus $A$ and $E_0$. With increasing $A$, there is a substantial deviation from the superposition-model predictions, which can apparently be attributed to nonlinear effects associated with both the possibility of a group suppression of muon production, when the heavy nucleus manages to reach a large depth in the atmosphere, and the production of a large number of muons, when an extensive air shower is nucleated by a heavy nucleus at a large height. Figure 2 shows the distribution in the number of muons of events generated by an iron nucleus with an energy

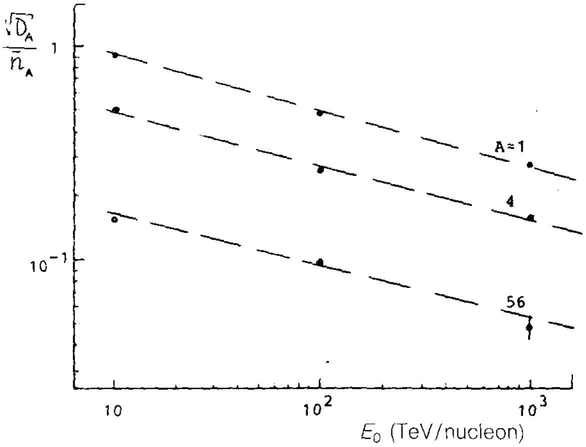

FIG. 1. Relative fluctuations in the multiplicity of muons with energies $E_\mu \geqslant 0.2$ TeV versus the atomc weight and energy per nucleon.

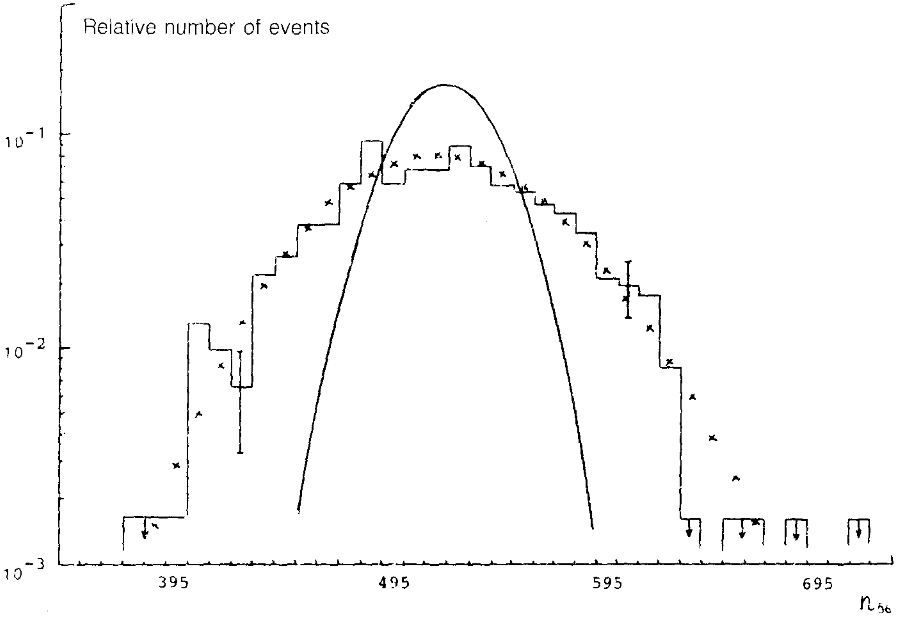

FIG. 2. Multiplicity distribution of muons with energies $E_\mu \geqslant 0.2$ TeV generated by an iron nucleus with an energy of 100 TeV/nucleon. Histogram—calculated; solid curve—Poisson distribution; x—negative binomial distribution.

$E_0 = 10^2$ TeV/nucleon. Shown for comparison is a calculated histogram with a Poisson distribution. We see that the Monte Carlo results cannot be described by a Poisson distribution but are described correctly by a negative binomial distribution[8,9] with a parameter $k = 135.5$.

In summary, we have found that for the mean multiplicities and mean muon production heights the superposition model holds very accurately for $\alpha$ particles and holds within an error $\delta < 3\%$ for iron nuclei over the primary-energy range $E_0 = 10$–$10^3$ TeV/nucleon. In contrast, we find a substantial deviation from the superposition-model predictions in terms of the measure of the dispersion of the distribution in $n_A$.

We note in conclusion that an experimental test of the superposition model will require knowledge of the characteristics of the primary nucleus, but such knowledge is possible only in accelerator experiments. Research is presently being carried out at CERN on $^{16}O$ and $^{32}S$ interactions at $E_0 = 0.2$ TeV/nucleon with various nuclei (see, for example, Refs. 10 and 11). This basic thrust of this research is to search for a quark-gluon plasma. Nucleus-nucleus interactions which satisfy the superposition model constitute a background in a search for a quark-gluon plasma and for other exotic phenomena associated with collective interactions of nucleons of interacting nuclei. From this standpoint, and also for the purpose of extracting information about the chemical composition of the high-energy primary cosmic rays, the superposition model is of practical interest, and it is an important matter to find further support for its validity.

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