Stopping of neutrinos in a material medium

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A universal upper limit is found on the energy loss of neutrinos in an arbitrary equilibrium medium (which is assumed for simplicity to be cold, nonrelativistic, homogeneous, and isotropic). This upper limit rules out the possibility of an anomalously pronounced stopping of neutrinos.

A natural scale value of the energy loss per unit time of neutrinos in a medium, Q, is given by the "collisional limit"

$$Q_0 = n\int_0^{\omega_0} d\omega\,\omega\, d\sigma/d\omega\ , \qquad (1)$$

where σ is the cross section for the scattering of a neutrino by a particle of the medium which is at rest, $\omega_0 = 2E^2/(2E+m)$ is the greatest energy transfer to the particle, m

and n are the mass and density of the particles of the medium, and E is the energy of the neutrino in its rest frame.

The important question of whether the inequality $Q \gg Q_0$ is permissible is still far from clear: The affirmative assertions which have been made contradict the opposite situation in the case of a charged particle, whose loss is always smaller than Q_0 (the Fermi density effect[1,2]). In this letter we demonstrate that the negative answer to this question is correct and that the inequality $Q \lesssim Q_0$ must hold for neutrinos.

1. We choose the interaction of a massless two-component neutrino with an electron of the medium in the standard four-fermion form[3]:

$$-\frac{G}{\sqrt{2}}\, j^{\mu} C_a J^a_{\mu} \,,$$

where j is the $(V-A)$ current of the neutrino, J^a is the vector $(a=V)$ or axial $(a=A)$ current of the electron, $C_V = 2\sin^2\theta_W + 1/2$, $C_A = \pm 1/2$ (the upper sign corresponds to the electron neutrino, and the lower one to the muon neutrino), θ_W is the Weinberg angle, G is the Fermi constant, and $\hbar = c = 1$. The interaction with the nucleon can be described in a corresponding way.

In second-order perturbation theory in G we have

$$Q = \frac{G^2}{4\pi^2 E^2} \int_0^{4E^2} dt \int_0^{1/\bar{\omega}} d\omega\, \omega\, S \,, \tag{2}$$

$$S = \frac{1}{4} \operatorname{tr} [(1+\gamma_5)\hat{p}\gamma^{\mu}(\hat{p}-\hat{k})\gamma^{\nu}]\, C_a C_b \operatorname{Im} D^{ab}_{\mu\nu} \,, \tag{3}$$

where ω and $-t$ are respectively the zeroth component and the square of the 4-momentum transferred to the medium, k_μ;p_μ is the 4-momentum of the neutrino; and $\bar{\omega} = E - t/4E$. The Green's function of the electron currents,

$$D^{ab}_{\mu\nu} = i\theta(x_0)\langle [J^a_\mu(x),\, J^b_\nu(0)]\rangle \,, \tag{4}$$

has the explicitly covariant parametrization

$$\left.\begin{aligned} D^{aa}_{\mu\nu} &= -R^a_1 g_{\mu\nu} + R^a_2 u_\mu u_\nu + \dots \,, \\ D^{VA}_{\mu\nu} &= D^{AV}_{\mu\nu} = i\epsilon_{\mu\nu\rho\sigma} k^{\rho} u^{\sigma} R_3/2m \,, \end{aligned}\right\} \tag{5}$$

where $g_{\mu\nu}$ is the metric tensor, u_μ is the 4-velocity of the medium, and the ellipsis replaces some terms proportional to k_μ or k_ν which are unimportant because of the transverse nature of the trace in (3). In the rest frame of the medium we have

$$S = \operatorname{Im}\{C^2_a[tR^a_1 + (2E^2 - 2E\omega - t/2)R^a_2] + t(2E-\omega)C_V C_A R_3/m\} \,. \tag{6}$$

We see that the loss of the neutrino depends on five characteristics of the medium: $R^a_{1,2}$ and R_3 (instead of the two characteristics, $R^V_{1,2}$, for a charged particle): Since parity is not conserved, the axial response of the medium as well as its vector response are important.

2. Collisional limit (1), where σ is the νe scattering cross section,[3] is found from (2) and (6) when (4) is calculated in the model of a low-density free electron gas. Expression (1) does not describe collective—in the broad sense of the term—effects in the medium, whose contribution is small at large neutrino energies $\omega_0 \gg E_0$, where $E_0 \ll m$ is a characteristic energy of the particles of the medium (in the region of close collisions), and

$$Q = Q_0 \qquad (E \gg (m E_0)^{1/2}). \tag{7}$$

We can thus restrict the analysis to the region $E \ll m$, where

$$Q_0 = \frac{2G^2 nE^4}{3\pi m} (C_V^2 + 5C_A^2). \tag{8}$$

In this region we can omit from (6) the term with R_3, which is proportional to E/m (the matrix γ_5 appears in the first power in D^{VA}).

3. The quantities $R^a_{1,2}$, which determine the loss of neutrinos with $E \ll m$, have the following properties. First, by introducing in (4) an intermediate system of eigenfunctions of the Hamiltonian of the medium, and using (5), we can verify that the following inequalities hold:

$$(t + \omega^2)\mathrm{Im}R_2^a > t\,\mathrm{Im}R_1^a > 0 \qquad (\omega, t > 0). \tag{9}$$

Furthermore, Green's function (4), while disappearing at $x_0 < |\mathbf{x}|$ (relativistic causality), is analytic along with the quantities R in the upper ω half-plane at a fixed t (Ref. 2). As a result, we have the Leontovich relations

$$R(\omega, t) = R(\infty, t) + \frac{2}{\pi}\int_0^\infty d\zeta\zeta\,\mathrm{Im}R(\zeta, t)/(\zeta^2 - \omega^2 - i\delta\omega)$$

and the sum rules

$$\int_0^\infty d\omega\omega\,\mathrm{Im}R(\omega, t) = \frac{\pi}{2}\alpha(t), \tag{10}$$

where, in the limit $\omega \to \infty$, we have

$$R(\omega, t) = R(\infty, t) - \alpha(t)/\omega^2 + \dots$$

(simple causality, $R = 0$ at $x_0 < 0$, means that R is an analytic function of ω at a fixed $\mathbf{k}$; it also leads to the Kramers-Kronig relations).

4. Below we will use only the sum rules for the quantities R_2^a, which are determined by an equation that follows from (4):

$$D^{ab} = P^{ab} + e^2 P^{aV} D P^{Vb}, \tag{11}$$

where P is a polarization operator (an electron loop with vector or axial vertices), and D is the Green's function of a photon in the medium. The second term in (11) describes the polarization of the medium caused by the neutrino current. In the vector case we then find the expression

$$R_2^V = \frac{t^2}{4\pi e^2(t+\omega^2)}\left[\frac{t}{(t+\omega^2-\epsilon_t\omega^2)} - \frac{1}{\epsilon_l}\right],$$

which describes the loss of a charged particle.[2] Here ϵ_l (ϵ_t) is the longitudinal (transverse) dielectric constant of the medium, which has the asymptotic behavior $\epsilon_{l,t} = 1 - \omega_p^2/\omega^2$ as $\omega \to \infty$, where $\omega_p^2 = 4\pi ne^2/m$ is the square of the plasma frequency of the electrons of the medium. In the axial case, the corresponding dielectric constant has the same asymptotic behavior, and the second term in (11) is absent under the condition $E \ll m$ (see the end of §2). We thus find

$$\alpha_2^V = nt^2/[m(t+\omega_p^2)], \qquad \alpha_2^A = nt/m. \tag{12}$$

5. To find an upper limit on the neutrino loss, we increase the right side of (2), attempting to reduce it to sum rules (10), (12). Strengthening inequality (9) by substituting $2\omega E$ in place of ω^2 on its left side [$\omega < E$; see (2)], and combining this inequality with (6), we find

$$S < (2E^2 + {}^t/_2)\, C_a^2 \,\mathrm{Im}\, R_2^a .$$

As we continue to increase the right side of (2), we extend the integration over ω in this expression to the entire positive semiaxis. It thus becomes possible to use the sum rules.

As a result, the upper limit on the neutrino loss is determined by the inequality

$$Q < \frac{2G^2nE^4}{3\pi m}\left[C_V^2\,\varphi(\omega_p^2/4E^2) + 5C_A^2\right], \qquad (E \ll m), \tag{13}$$

where the function

$$\varphi(x) = 6\int_0^1 dz z^2(1+z)/(z+x)$$

is 5 for $x = 0$ and falls off monotonically to zero with increasing x. The decrease in the loss which results from this function is of the same nature as the density effect for the loss of a charged particle: a suppression of the contribution of remote ($t < \omega_p^2$) collisions by the medium [see (12)].

From inequalities (7) and (13), combined with (8), we can draw the conclusion that the energy loss by a neutrino is less than or on the order of the collisional limit.

A detailed version of this letter and also a generalization of its results to crystalline, hot, relativistic media will be published separately.

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[2]L. A. Kirzhnits, Pis'ma Zh. Eksp. Teor. Fiz. **46**, 244 (1987) [JETP Lett. **46**, 308 (1981)].
[3]L. B. Okun', *Leptons and Quarks*, North-Holland, Amsterdam, 1981.

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