Effective two-loop potential for SU(2) and SU(3) gluodynamics at a nonzero temperature

V. M. Belyaev and V. L. Eletskiĭ
Institute of Theoretical and Experimental Physics

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The effective potential $W(A_0)$ for SU(2) and SU(3) gluodynamics has been calculated by the two-loop method for $T \neq 0$; here A_0 is the temporal component of the gauge field. The minimum of $W(A_0)$ is reached at $A_0 \neq 0$. At the two-loop level, the SU(2) symmetry is broken to U(1), and SU(3) is broken to U(1) × U(1).

Gauge theories for $T \neq 0$ are presently attracting considerable interest in connection with experiments on the collisions of ultrarelativistic heavy ions,[1] in which the production of a hot quark-gluon plasma is expected. These problems have been the subject of many reviews, with extensive bibliographies (e.g., Refs. 2 and 3). It is usually assumed that the effective coupling constant $g(T)$ is small by virtue of asymptotic freedom and that perturbation theory can be used. It turns out,[2,4] however, that perturbative calculations run into the difficulty of infrared infinities, which unavoidably arise even at a fairly low order of the theory. It is expected that these infinities, which stem from zero modes of the spatial components of the gauge field A_i, can be effectively cut off at momenta on the order of g^2T as a result of the appearance of a so-called magnetic mass: an effective mass of the fields A_i.

There is an important distinction between a gauge theory at $T \neq 0$ and one at $T = 0$: At $T = 0$, a gauge transformation can be used to change a constant field A_0 by an arbitrary constant; in particular, it can be caused to vanish. In other words, a theory with $A_0 \neq 0$ is physically equivalent to one with $A_0 = 0$. At $T \neq 0$, in the Euclidean formulation of the theory, the temporal component becomes compact, $0 \leqslant x_0 \leqslant \beta$, where $\beta = 1/T$, and the field A_μ satisfies periodic boundary conditions, $A_\mu(0) = A_\mu(\beta)$. Now a zero mode $A_0 = \text{const}$ cannot, in general, be changed in gauge while the periodic boundary conditions are retained for the spatial components of A_i. In the SU(2) case, for example, gauge shifts of A_0 by a constant which is a multiple of $2\pi/\beta g$ are permissible, and a quantization near $A_0 \neq 0$ is generally not physically equivalent to the case $A_0 = 0$. Several recent papers[5–9] show indications that a field condensate A_0 arises at $T \neq 0$ in a non-Abelian theory.

We have carried a two-loop calculation of $W(A_0)$, the effective potential of the field A_0, for the case of SU(2) and SU(3) gluodynamics. This calculation confirms the existence of a nontrivial minimum of $W(A_0)$, which breaks the gauge symmetry at high temperatures at the two-loop level. The calculations were carried out in the Euclidean formulation of the theory, in the external-field formalism. In the calculations of $W(A_0)$ we used the Feynman background gauge $D^b_\mu A^a_\mu = 0$, where $(D^b_\mu)^{ab} = \partial_\mu \delta^{ab} + g f^{abc}(A^c_\mu)^b$, and we have the background field $(A^a_\mu)^b = (A^3_0)^b \delta_{\mu 0} \delta^{a3} = \text{const}$ in the SU(2) case and

$(A^a_\mu)^b = (A^3_0\delta^{a3} + A^8_0\delta^{a8})^b\delta_{\mu 0}$ in the SU(3) case. After the ghosts required for the retention of gauge invariance are added, the Lagrangian of the theory becomes

$$\mathcal{L} = \frac{1}{4} G^2_{\mu\nu} + \frac{1}{2\alpha} (D^b_\mu A_\mu)^2 + \bar{\chi} D^b_\mu D_\mu \chi, \tag{1}$$

where

$$A^a_\mu = (A^a_\mu)^q + (A^a_\mu)^b,\ G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$
$$= D^b_\mu (A^a_\nu)^q - D^b_\mu (A^a_\mu)^q - g f^{abc} (A^b_\mu)^q (A^c_\nu)^q, \text{ and } \alpha$$

is the gauge parameter. We see that the constant external field lengthens the ordinary derivative in the definition of $G_{\mu\nu}$. It is convenient to transform to a "meson" basis: $\pi^0_\mu = A^3_\mu$, $\pi^\pm_\mu = (A^1_\mu \pm iA^2_\mu)/\sqrt{2}$, $K^\pm_\mu = (A^4_\mu \pm iA^5_\mu)/\sqrt{2}$, $K^0_\mu = (A^6_\mu + iA^7_\mu)/\sqrt{2}$, $\bar{K}^0_\mu = (A^6_\mu - iA^7_\mu)/\sqrt{2}$, $\eta_\mu = A^8_\mu$. Here we have written the field for the SU(3) case. This particular basis was chosen because of the analogy with the meson octet; the corresponding fields have the same charges with respect to the external fields A^3_0 and A^8_0 as the third component of the isospin and the hypercharge of the corresponding mesons. In the SU(2) case, we considered only the fields π_μ. A corresponding basis is used for the ghosts. In this basis the quadratic part of the Lagrangian is diagonal. A shift occurs in the sums over frequencies $k_0 = 2\pi n/\beta \rightarrow k^{(i)}_0 = 2\pi n/\beta + C_i$, where $C_i = \pm g A^3_0, \pm g/2\ (A^3_0 \pm \sqrt{3} A^8_0)$ for the $\pi^\pm$, $K^\pm$ and $K^0(\bar{K}^0)$ fields. There is no such shift for the "neutral" π^0 and η fields: $k^0_0 = 2\pi n/\beta$. The frequency shift occurs not only in the propagator but also at the vertices. This circumstance was overlooked in Ref. 5, with an incorrect conclusion as a result. A detailed exposition of the calculation procedure in the case of the SU(2) group is given in Ref. 9.

The single-loop contribution to WA_0 is given by

$$W^{(1)}_{SU(2)}(x) = \frac{4\pi^2}{3\beta^4}\left[-\frac{1}{60} + B_4(x/2)\right], \tag{2}$$

$$W^{(1)}_{SU(3)}(x, y) = \frac{4\pi^2}{3\beta^4}\left[-\frac{1}{30} + B_4(x/2) + B_4((x + \sqrt{3}y)/4) + B_4((x - \sqrt{3}y)/4)\right],$$

where $x = gA^3_0\beta/\pi$, $y = gA^8_0\beta/\pi$, and $B_4(x) = x^2(1-x)^2 - 1/30$ is the Bernoulli polynomial. The arguments in Bernoulli polynomials are defined modulo 1. The minima of the single-loop effective potentials in (2) are reached at $x = 2\pi$ in the SU(2) case and at $x = 2n$, $y = 2(2m - n)/\sqrt{3}$ in the SU(3) case ($m, n = 0, \pm 1, \pm 2, \ldots$). These expressions agree with the results derived by Weiss.[10]

The two-loop calculation leads to the following contributions to W:

$$W^{(2)}_{SU(2)} = \frac{g^2}{2\beta^4}\left[B^2_2(x/2) + 2B_2(0)(B_2(x/2)\right]$$

$$W^{(2)}_{SU(3)}(x,y)=\frac{g^2}{2\beta^4}[B_2^2(x/2)+B_2^2((x+\sqrt{3}y)/4)+B_2^2((x-\sqrt{3}y)/4)+2B_2(0)(B_2(x/2)$$

$$+B_2((x+\sqrt{3}y)/4)+B_2((x-\sqrt{3}y)/4))+B_2(x/2)B_2((x+\sqrt{3}y)/4)$$

$$+B_2(x/2)B_2((x-\sqrt{3}y)/4)$$

$$+B_2((x+\sqrt{3}y)/4)B_2((x-\sqrt{3}y)/4)], \qquad (3)$$

where $B_2(x)=1/6-x(1-x)$, and the arguments of Bernoulli polynomials are again defined modulo 1.

Since the potentials are periodic in the corresponding arguments of the effective potentials—this circumstance corresponds to an invariance of the theories under gauge transformations of the centers of the groups—it is sufficient to consider the region near one minimum of effective potentials (2) with $x=0$ for SU(2) and $x=0$, $y=0$ for SU(3). In this region the minima are reached at the two-loop level at $x=\pm g^2T/4\pi$ in the SU(2) case and at $(x=\pm g^2T/2\pi,\ y=0)$, $(x=\pm g^2T/4\pi$, $y=\pm\sqrt{3}g^2T/4\pi)$ and $(x=\pm g^2T/4\pi, y=\mp\sqrt{3}g^2T/4\pi)$. In the SU(3) case. All the minima are physically equivalent. The existence of such nontrivial minima leads to a breaking of the gauge symmetries [SU(2) to U(1), and SU(3) to U(1)×U(1)]. The zero modes of the spatial components of the gauge fields, which are of a nonsinglet nature with respect to the background field, acquire a mass at the tree level: $m=g^2T/4\pi$ in the SU(2) case and $m_\pi=g^2T/2\pi$, $m_K=g^2T/4\pi$ in the SU(3) case (in the case of a minimum, $x=g^2T/2\pi$, $y=0$). The spatial components of the gauge fields, which are of a singlet nature with respect to the background field, remain massless. The fields A_0 acquire[2,3] a so-called Debye mass at the single-loop level. This electrostatic mass is reproduced from the effective potential which has been found:

$$m_D^2=\Pi_{00}(k=0)=\partial^2/\partial(A_0^b)^2[W^{(1)}(A_0^b)+W^{(2)}(A_0^b)]$$

$$=\frac{g^2}{\pi^2T^2}\partial^2/\partial x^2[W^{(1)}(x)+W^{(2)}(x)]=\begin{cases}2g^2T/3-SU(2)\\ g^2T\quad -SU(3)\end{cases}. \qquad (4)$$

The conclusion that a condensate of the field A_0 exists in perturbation theory requires a comprehensive test. There is the possibility in principle that the contributions of higher orders of perturbation theory will be on the order of the two-loop contribution at $x\approx g^2$. In this case we could not draw a conclusion about the size of the condensate or even about its existence. At the n-loop level, we could draw such conclusions if $W^{(n)}$ contained a term $\alpha g^{2(n-1)}/x^{n-3}$. However, at $x\sim g^2$ terms of this type would have made a contribution on the order of gT to the Debye mass, according to expression (4). We thus have two possibilities: Either these terms are absent from W, or problems concerning the Debye mass arise in higher orders of perturbation theory.

It is clear from the discussion above that the infrared structure of the effective potential under discussion here requires study, not only for testing the conclusion regarding the appearance of a condensate but also for refining the status of the Debye mass.

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