Covariant action for chiral boson strings

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A covariant action is proposed for chiral boson strings with d space-time dimensions, with N_L left-hand moving bosons, N_R right-hand moving bosons, and $2N_F$ nonpropagating Majorana-Weyl fermions. The theory is free of anomalies in the case $N_R = N_L = 26 - d, N_F = 2d$.

A covariant quantization of models with chiral bosons has recently been the subject of a wide discussion. Interest in such models stems from the well-known fact that there are two formulations of heterotic strings, in $d = 10$ (Ref. 1) and $d = 4$ (Ref. 2), in which the internal degrees of freedom are realized by either chiral bosons or chiral fermions. Furthermore, chiral bosons must be introduced for an explicitly supersymmetric description of four-dimensional strings.[3,4]

A covariant action which describes the classical mechanics of chiral bosons was proposed several years ago by Siegel.[5] In Siegel's approach, the chirality condition is taken into account through the introduction in the action of a constraint with a corresponding Lagrange multiplier. In the process, a new symmetry (Siegel symmetry), analogous to diffeomorphisms, arises. At the quantum level, however, this symmetry

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is anomalous; the conditions under which the anomaly cancels out do not agree with a canonical analysis. For example, a canonically quantized boson string with d space-time coordinates, N_L left-hand moving bosons, and N_R right-hand moving bosons, is free of anomalies in the case $N_L = N_R = 26 - d$, while a cancellation of the anomaly of Siegel symmetry occurs at $N_L = N_R = 26$ and thus $d = 0$.

In an effort to resolve this problem, Hull[6] has proposed modifying the action of chiral bosons by adding to Siegel's action a new term which depends on classically nonpropagating fields. These additional fields acquire a kinetic term at the quantum level and can be used to cancel the anomaly of the Siegel symmetry.

In the present letter we propose a new chiral string action, in which a nondynamic sector is realized in terms of nonpropagating spinors. We find the conditions for the cancellation of anomalies, and we prove that our model is equivalent to Hull's. Our notation is the same as that of Ref. 6.

Let us consider the action for a chiral boson string with d space-time coordinates X^P, N_L left-hand bosons Y^i, N_R right-hand bosons Z^m, N_F fermions Ψ^I_-, and N'_F fermions φ^M_+:

$$S = - \int d^2\sigma e \{\eta_{pq} \nabla_= X^p \nabla_{\ddagger} X^q + D_{\ddagger} Y^i D_= Y^i$$

$$+ \bar{D}_{\ddagger} Z^m \bar{D}_= Z^m + \frac{i}{2}\Psi^I_- D_{\ddagger} \Psi^I_- + \frac{i}{2}\varphi^M_+ \bar{D}_= \varphi^M_+ \} , \qquad (1)$$

where $\beta_{pq} = \mathrm{diag}(-1, +1, \ldots, +1)$, and the covariant derivatives are defined by

$$D_{\ddagger} = \nabla_{\ddagger} - \Lambda_{\ddagger\ddagger} \nabla_= + (\nabla_= \Lambda_{\ddagger\ddagger})\hat{M}, \qquad D_= = \nabla_= ,$$

$$[D_= , D_{\ddagger}] = \frac{1}{2} R^Y \hat{M}, \qquad R^Y = R + 2\nabla_= \nabla_= \Lambda_{\ddagger\ddagger} ,$$

$$\bar{D}_{\ddagger} = \nabla_{\ddagger} , \qquad \bar{D}_= = \nabla_= - \Lambda_{==} \nabla_{\ddagger} - (\nabla_{\ddagger} \Lambda_{==})\hat{M},$$

$$[\bar{D}_= , \bar{D}_{\ddagger}] = \frac{1}{2} R^Z \hat{M}; \qquad R^Z = R + 2\nabla_{\ddagger} \nabla_{\ddagger} \Lambda_{==} .$$

Here $\nabla_{\neq\neq}$ and $\nabla_{==}$ are covariant derivatives which are associated with the tetrad e^α_μ on the world sheet, and $\hat{M}$ is a Lorentz generator. The Lagrange multipliers $\Lambda_{\neq\neq}$ and $\Lambda_{==}$ are gauge fields for Siegel symmetry.[5] From the equations of motion for $\Lambda_{\neq\neq}$, Ψ^I follow the relations $\nabla_= Y^i = \Psi^I_- = 0$. In a corresponding way, one can show that we have $\nabla_{\neq} Z^m = \varphi^M_+ = 0$. Consequently, action (1) describes the classical dynamics of a chiral boson string.

To quantize the model, we use the background-field method. The contribution to the effective action Γ_{eff} from the variables X^p is given by the well-known expression

$$\Gamma_X = - \frac{d}{96\pi} \int d^2\sigma e R \frac{1}{\Box} R, \qquad \Box \equiv \{ \nabla_{\ddagger} , \nabla_= \} . \qquad (3)$$

Correspondingly, the contributions to the effective action from Y^i and Z^m are

$$\Gamma_Y + \Gamma_Z = -\frac{N_L}{96\pi} \int d^2\sigma e R^Y \frac{1}{\Box_Y} R^Y - \frac{N_R}{96\pi} \int d^2\sigma e R^Z \frac{1}{\Box_Z} R^Z. \quad (4)$$

The ghost part of the effective action is constructed in the following way:

$$\Gamma_{Gh} = \frac{26}{96\pi} \int d^2\sigma e \{R^Y \frac{1}{\Box_Y} R^Y + R^Z \frac{1}{\Box_Z} R^Z\}. \quad (5)$$

In the case $N_F = N'_{F'}$ the effective action for fermions can be written in the covariant form

$$\Gamma_{\Psi,\varphi} = -\frac{N_F}{192\pi} \int d^2\sigma e \{ R^Y \frac{1}{\Box_Y} R^Y + R^Z \frac{1}{\Box_Z} R^Z - R \frac{1}{\Box} R \}. \quad (6)$$

The complete effective action Γ_{eff} of model (1) ($N_F = N'_F$) is the sum of expressions (3)–(6). A variation of Γ_{eff} with respect to Λ vanishes under the conditions

$$N_L + \frac{1}{2} N_F = 26, \qquad N_R + \frac{1}{2} N_F = 26. \quad (7)$$

In turn, a cancellation of the Weyl anomaly occurs if the following relation holds:

$$2d = N_F. \quad (8)$$

Equations (7) and (8) show that the model is free of anomalies if and only if a canonical quantization of the corresponding theory is justified.

We conclude with the question of the equivalence of the theory with action (1) to Hull's string theory.[6] In Hull's approach, the bosons W^α described by the action

$$S_H = - \int d^2\sigma E D_{\neq} W^\alpha D_= W^\alpha, \qquad E \equiv e(1 - \Lambda^2), \quad (9)$$

$$D_{\neq} = (1 - \Lambda^2)^{-1} D_{\neq}, \quad D_= = (1 - \Lambda^2)^{-1} \bar{D}_=, \quad \Lambda^2 \equiv \Lambda_{\neq\neq} \Lambda_{==} < 1,$$

are nondynamic fields.

It can be shown that the effective action Γ_W corresponding to the theory in (9) agrees within finite local counterterms with (6) in the case $N_W = 2N_F$. Action (9) is thus equivalent to the action of fermions in (1).

In our approach, however, the restriction $\Lambda^2 < 1$ does not arise. That restriction makes the quantization procedure of theory (9) somewhat unsatisfactory. Furthermore, the formulation which we have proposed here is the most convenient one for a supersymmetric generalization, in particular, for describing four-dimensional superstrings.[2]

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