Power-law corrections to parton sum rules for deep inelastic scattering by polarized target

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(Submitted 21 June 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **50**, No. 2, 54–57 (25 July 1989)

The second moment of the structure function $g_2(x)$ and power-law corrections to the integral of the structure function $g_1(x)$ are calculated on the basis of the QCD sum rules.

1. The EMC data[1] on deep inelastic scattering by a polarized target reveal a "spin crisis," which has yet to find a satisfactory resolution. In the scaling limit $Q^2 \to \infty$, the structure function $g_1(x)$ alters the spin distribution in a hadron among its constituents.[2,3] The value $\int dx g_1^p(x,Q^2) \approx 0.114$ (Ref. 1), which is smaller than the value expected, implies that a substantial fraction of the proton spin is carried by gluons, while the contribution of valence quarks is small.

The importance of this assertion makes it worthwhile to examine possible nonscaling $1/Q^{2n}$ corrections to the parton sum rules. Their role has yet to be resolved. Anselmino *et al.*[4] have suggested that a large part of the discrepancy between the experimental result and the predictions of the standard model is due to specifically large power-law corrections. On the other hand, there are several indirect arguments against an important role of nonscaling corrections (the data depend only weakly on Q^2).

In this letter we report the results of a new calculation of the $1/Q^2$ power-law corrections to the quantity $\int dx g_1(x,Q^2)$ by the method of QCD (quantum chromodynamics) sum rules. At the characteristic values $Q^2 \sim 10$ GeV^2, these corrections turn out to be extremely small, incapable of explaining the spin crisis and apparently not necessary to consider in an analysis of experimental data. The contributions of the nonleading twist 3 to the second moment $[\int dx x^2 g_2(x)]$ of the structure function $g_2(x,Q^2)$ were calculated as part of the overall $1/Q^2$ power-law correction. These contributions are not small in comparison with those of the leading-twist operators.

There are plans to carry out experiments to measure the structure function $g_2(x)$, so this result is of interest in its own right.

2. Our analysis starts from the results of Ref. 5 on the operator expansion of the amplitudes for the deep inelastic scattering of leptons by polarized nucleons, which lead to the sum rules

$$\int dx g_1^{(p\pm n)}(x,Q^2) = \binom{5/18}{1/6}\{g_A^{S(NS)}(1-\frac{\alpha_s(Q^2)}{\pi}) - \frac{8}{9Q^2}\ll O^{S(NS)}\gg\}$$

$$+\frac{8}{3}\frac{m_N^2}{Q^2}\int dx x^2(g_2^{(p\pm n)}(x)+\frac{5}{6}g_1^{(p\pm n)}(x)) + O(1/Q^4). \qquad (1)$$

Here and below, the upper number in parentheses means the sum of, and the lower number the difference between, the cross sections for the proton and the neutron, and $\langle\langle O^{S(NS)}\rangle\rangle$ are the reduced matrix elements in terms of the proton from local twist-4 operators:

$$O_\mu^{S(NS)} = \bar{u}_g\tilde{G}^a_{\mu\nu}\gamma_\nu\frac{\lambda^a}{2}u \pm (u\to d),$$

$$\langle N|O_\mu|N\rangle = s_\mu\ll O\gg, \qquad s_\mu = \bar{u}_p\gamma_\mu\gamma_5 u_p\ . \qquad (2)$$

The contributions of strange quarks to $\langle\langle O^S\rangle\rangle$ can be ignored. The axial constants are $g_A^{NS}=1.25$, $g_A^S=\sqrt{3}/5(a_s+2\sqrt{2}a_0)$, $\sqrt{3}a_s=3F-D$.

The term $\sim m_N^2$ on the right side of (1) is the so-called kinematic power-law correction. It contains a contribution of another structure function, $g_2(x)$. As we know, g_2 differs from g_1 in that it contains twist-2 and twist-3 operators on an equal footing. It is convenient to separate them. In particular, we can write[5]

$$\int x^2 dx g_2^{p\pm n}(x) = -\frac{2}{3}\int x^2 dx g_1^{p\pm n} - \frac{1}{6}\binom{5/18}{1/6}\ll Q^{S(NS)}\gg, \qquad (3)$$

where

$$Q^{S(NS)}_{\mu\nu,\sigma} = S_{\nu,\sigma}\bar{u}_g\tilde{G}^a_{\mu\nu}\gamma_\sigma\frac{\lambda^a}{2}u \pm (u\to d) - \text{traces},$$

$$\langle N|Q_{\mu\nu,\sigma}|N\rangle = S_{\nu,\sigma}A_{\mu,\nu}s_\mu p_\nu p_\sigma\ll Q\gg. \qquad (4)$$

Here $A_{\mu,\nu}$ and $S_{\nu,\sigma}$ are (anti-) symmetrizers in terms of the corresponding indices.

It is worthwhile to rewrite (1), retaining only contributions $\sim g_1(x)$ as a kinematic correction, while incorporating the twist-3 contributions to $g_2(x)$ in a dynamic correction [in braces (curly brackets) in (1)]. It is not difficult to see that the latter takes the form

$$-\frac{8}{9Q^2}(\ll O\gg + \frac{1}{2}m_N^2\ll Q\gg).$$

The problem is to calculate the matrix elements $\langle\langle O\rangle\rangle$ and $\langle\langle Q\rangle\rangle$.

3. For this purpose we examine the three-point correlation functions of operators (2), (4) with a nucleon current $\eta = \epsilon^{abc}(u^a C\gamma_\lambda u^b)\gamma_5\gamma_\lambda d^c$:

$$i^2\int dx\, e^{ipx}\int dy\langle T\{\eta(x)O_\mu(y)\bar{\eta}(0)\}\rangle = -2p_\mu \hat{p}\gamma_5\lambda_p^2 \langle\langle O\rangle\rangle (m_N^2 - p^2)^{-2} + \dots \quad (5)$$

$$i^2\int dx\, e^{ipx}\int dy\langle T\{\eta(x)\, Q_{\mu\nu,\sigma}(y)\bar{\eta}(0)\}\rangle$$
$$= -2S_{\nu,\sigma}A_{\mu,\nu}p_\sigma p_\mu\gamma_\nu\gamma_5 m_N^2\langle\langle Q\rangle\rangle (m_N^2 - p^2)^{-2} + \dots, \quad (6)$$

where λ_p is the coupling constant of the coupling of the proton with the current η. The operator expansions for the correlation functions (5), (6) take the following forms, respectively, in the Euclidean region, $p^2 \to -\infty$:

$$p_\mu\hat{p}\gamma_5\Big\{\begin{pmatrix}9\\5\end{pmatrix}\frac{\alpha_S}{360\pi^5}p^4\ln^2(\mu^2/-p^2)$$
$$-\begin{pmatrix}0\\1\end{pmatrix}\frac{\langle\frac{\alpha_S}{\pi}G^2\rangle}{72\pi^2}\ln(\mu^2/-p^2) - \begin{pmatrix}1\\0\end{pmatrix}\frac{f_\pi^2\delta^2}{3\pi^2}\ln(\mu^2/-p^2)$$
$$+\begin{pmatrix}\ln(\mu^2/-p^2) - 1/24\\ \ln(\mu^2/-p^2) + 11/24\end{pmatrix}\frac{16\alpha_S}{27\pi}\frac{\langle\bar{\psi}\psi\rangle^2}{-p^2} + \frac{\Pi}{36\pi^2}\frac{1}{-p^2} - \begin{pmatrix}3\\1\end{pmatrix}\frac{m_0^2\langle\bar{\psi}\psi\rangle^2}{9p^4} + \dots\Big\}, \quad (7)$$

$$2S_{\nu,\sigma}A_{\mu,\nu}p_\sigma p_\mu\gamma_\nu\gamma_5\Big\{\begin{pmatrix}7\\-13\end{pmatrix}\frac{\alpha_S}{4320\pi^5}p^4\ln^2(\mu^2/-p^2) - \begin{pmatrix}0\\1\end{pmatrix}\frac{\langle\frac{\alpha_S}{\pi}G^2\rangle}{72\pi^2}\ln(\mu^2/-p^2)$$
$$-\begin{pmatrix}2\ln(\mu^2/-p^2) - 71/6\\ 20\ln(\mu^2/-p^2) + 157/6\end{pmatrix}\frac{\alpha_S}{27\pi}\frac{\langle\bar{\psi}\psi\rangle^2}{-p^2}$$
$$+\begin{pmatrix}-1\\2\end{pmatrix}\frac{R}{144\pi^2}\frac{1}{-p^2} + \begin{pmatrix}1\\-1\end{pmatrix}\frac{m_0^2\langle\bar{\psi}\psi\rangle^2}{9p^4} + \dots\Big\},$$

where

$$\Pi = i\int dy\langle T\{O_\mu(y)O_\mu(0)\}\rangle \approx 3\times 10^{-3}\ \text{GeV}^6, \quad (8)$$

$$R = i\int dy\langle T\{Q_{\mu\nu,\sigma}(y)Q_{\mu\nu,\sigma}(0)\}\rangle \approx 1\times 10^{-3}\ \text{GeV}^6, \quad (9)$$

$\delta^2 = 0.2\ \text{GeV}^2$, and $m_0^2 = \langle\bar{\psi}\sigma G\psi\rangle/\langle\bar{\psi}\psi\rangle$. A detailed derivation of expansions (7) and (8) and of numerical estimates (9) will be given in a separate paper.

We now apply the standard technique of QCD sum rules (Ref. 6, for example). Using a Borel transformation, subtracting the various background contributions (including the single-pole terms), and equating expansions (5) and (7) and also (6) and

(8) in the region of values $M^2 \sim 7$ GeV^2 of the Borel parameter, we find

$$\ll O^{NS} \gg \approx 0.18\,\text{GeV}^2, \qquad \ll O^S \gg \approx -(0-0.06)\,\text{GeV}^2, \tag{10}$$

$$\ll Q^{NS} \gg \approx 0.28, \qquad \ll Q^S \gg \approx -0.20. \tag{11}$$

The error in the determination of each of matrix elements (10) and (11) is on the order of ± 0.1. We then find some rules (1) in the form

$$\int dx g_1^{p-n}(x, Q^2) = \frac{1}{6}\{g_A(1 - \frac{\alpha_S(Q^2)}{\pi}) - \frac{0.3\,\text{GeV}^2}{Q^2}\} + 4\frac{m_N^2}{Q^2}\int dx x^2 g_1^{p-n}(x), \tag{12}$$

$$\int dx g_1^{p+n}(x, Q^2) = \frac{5}{18}\{g_A^S(1 - \frac{\alpha_S(Q^2)}{\pi}) + \frac{0.1\,\text{GeV}^2}{Q^2}\} + 4\frac{m_N^2}{Q^2}\int dx x^2 g_1^{p+n}(x). \tag{13}$$

We believe that the error in the determination of the power-law correction to Bjorken sum rule (12) is no worse than $\sim 50\%$. The correction in the singlet channel is smaller than that in the nonsinglet channel and has the opposite sign. The error in this case is on the order of 100%. Note that the scale of the power-law corrections in (12) and (13) is about an order of magnitude below the expectations of Ref. 4.

The second moment, $\int dx x^2 g_1(x)$ is not yet known, since the measurements have been restricted to the region[1] $x < 0.5$. Adopting the crude model $g_1^{p-n} \sim \frac{1}{6} g_A \times 4(1-x)^3$ for an estimate, we find the kinematic correction to the Bjorken sum rule to be $+0.05(m_N^2/Q^2)$. This correction cancels out the dynamic correction for the interaction of quarks which we calculated above.

Our basic conclusion is that the power-law corrections to the parton sum rules are extremely small and are unimportant in interpreting data at this stage. The scale of these corrections is similar to that of the corrections to the Gross-Llewellyn-Smith sum rules calculated previously[6] for the integral of structure function F_3.

Substituting the calculated values from (11) into (3), and using the rough estimate $\frac{2}{3}\int dx x^2 g_1^{p-n} \sim 0.01$ for a comparison, we see that the contributions of twist 3 to $g_2(x)$ are not small. They differ in sign in the singlet and nonsinglet channels. This unexpected result will be amenable to an experimental test in the near future.

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Translated by Dave Parsons