Does a scaling theory describe the magnetoconductivity of silicon metal-insulator-semiconductor structures?

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It has been shown experimentally that the magnetoconductivity of silicon metal-insulator-semiconductor structures at $T < 1$ K is not described by the kinetic equation, which predicts that the maxima in σ_{xx} will increase linearly with the index of the Landau level. The results can be explained on the basis of a two-parameter scaling theory.

The question of the magnetoconductivity of 2D electron systems in the regions between Hall plateaus is not completely resolved. Experiments on heterostructures[1,2] support the two-parameter scaling theory,[3–5] according to which the σ_{xx} (σ_{xy}) phase diagrams and, in particular, the maxima of the diagonal component of the conductivity in the limit $T \rightarrow 0$ are the same for all Landau levels. On the other hand, results on silicon metal-insulator-semiconductor (MIS) structures (Refs. 6 and 7, for example) confirm that the maxima of σ_{xx} depend in a linear fashion on the index of the Landau level, N: $\sigma_{xx}^{\max} = (e^2/\pi^2\hbar)(N + 1/2)$. This behavior follows from a kinetic equation.[7]

In the present letter we attempt to resolve the contradiction which has arisen. For this purpose we have studied silicon MIS structures with a high electron mobility, in which energy sublevels of the zeroth and first Landau levels are completely or almost completely resolved at $T < 1$ K in magnetic fields $H > 10$ T. The results reported below were obtained from samples having the Hall geometry: Si 0-A [a maximum electron mobility $\mu_{max} = 3.5$ m^2/(V·s) at a density $n = 3.2\times10^{15}$ m^{-2}] and Si 2–10 [$\mu_{max} = 2.4$ m^2(V·s) at $n = 5.5\times10^{15}$ m^{-2}]. The measurements were taken at an alternating current of 0.1 μA with a frequency of 4 Hz. We used two phase detectors, which made it possible to simultaneously measure the components ρ_{xx} and ρ_{xy} of the magnetoresistance tensor and then to calculate the magnetoconductivity from these results.

Figure 1 shows σ_{xx} versus the gate voltage V_g for the zeroth and first Landau levels (we are not reporting the results for higher levels here since the valley splitting was nearly unresolved for them). It can be seen from this figure that at $T\approx2$ K the maxima of the conductivity σ_{xx}^{max}, which correspond to half-integer filling factors ν, are approximately three times as high for $N = 1$ as for $N = 0$. Within a single Landau level, the values of σ_{xx}^{max} are nearly the same for the various sublevels.[1)] This quantitative relation between the values of σ_{xx}^{max} for the zeroth and first Landau levels would seem at first glance to correspond to the results found[7] through a solution of a kinetic equation. This agreement with Ref. 7 (which has also been mentioned in several previous papers[6,7]), however, disappears as the temperature is lowered: The maxima of σ_{xx} for $N = 1$ decrease, approaching those for $N = 0$. This behavior of the magnetoconductivity at the first Landau level is characteristic of all the high-mobility MIS structures which we studied. A decrease in σ_{xx}^{max} ($N = 1$) can also be seen in Fig. 13 of Ref. 9, although this effect was not discussed there. For $N = 0$ the variations in σ_{xx}^{max} over the temperature interval studied are small; a slight decrease ($\approx10\%$) in the conductivity is observed at $T < 0.6$ K.

In the semiclassical region, $T > T_{sc} \approx \Gamma/K_B$ (Γ is the energy-level width caused

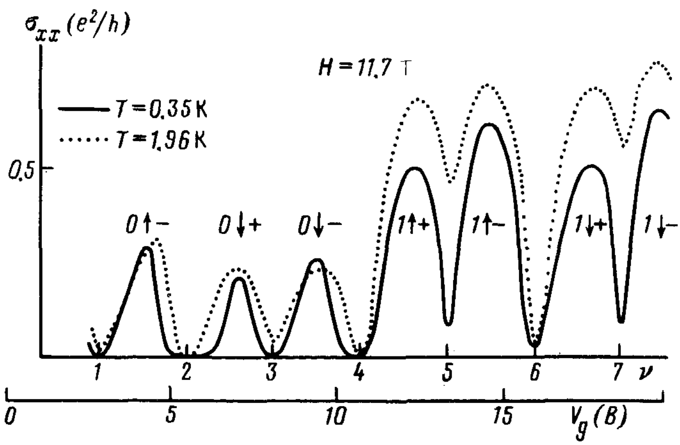

FIG. 1. $\sigma_{xx}(\nu)$ for sample Si 0-A at $H = 11.7$ T and at two temperatures.

by elastic collisions of electrons, and K_B is the Boltzmann constant), a decrease in the temperature could be accompanied by only an increase in σ_{xx}^{max} due to a contraction of the Fermi distribution. It can thus be suggested that the variations in the magnetoconductivity at $T<0.6$ K ($N=0$) and $T<2$ K ($N=1$) are of a quantum-mechanical nature and are caused by a change in the length scale of the system—in this case the distance L_ϕ, over which the phase coherence of the electrons is lost. Another reason for a decrease in σ_{xx}^{max} might be an electron-electron interaction,[10] but in this case the effect should have been manifested to a greater extent in samples with a relatively low electron mobility, while experimentally the correlation is more likely the opposite.

According to the scaling theory,[4,5] the "starting" points on the phase paths in the $(\sigma_{xx}, \sigma_{xy})$ plane correspond to $L_\phi = l_H$ (l_H is the magnetic length) and are determined from a kinetic equation which predicts the curves of $\sigma_{xx}(\sigma_{yx})$ shown by the dashed lines in Fig. 2 for $T=0$ and for short-range scatterers. The changes in σ_{xx} caused by the changes in L_ϕ are described by the solution of the equation

$$\partial\sigma'_{xx}/\partial\ln(L_\phi/l_H) = \beta_{xx}(\sigma'_{xx}, \sigma'_{xy}),$$

where $\sigma'_{ij} = \sigma_{ij}/(e^2/h)$, $\beta_{xx} \approx -(1/8\pi^2\sigma'_{xx}) - D^0\sigma'^3_{xx}\cos(2\pi\sigma'_{xy})\exp(-4\pi\sigma'_{xx})$, and D^0 is a universal positive constant, which is equal to $(4\pi)^2$ for a Gaussian potential. The points at which the phase paths terminate as $L_\phi \to \infty$ should be the points $(ne^2/h, 0)$ (where n is an integer) for all initial values of σ_{yx} other than half-integer values, for which the phase paths should terminate at points $(ne^2/2h, \sigma^0)$. The value of σ^0 is found from the condition $\beta_{xx}(\sigma_{xx}, \sigma_{xy} = ne^2/2h) = 0$ and depends on D^0.

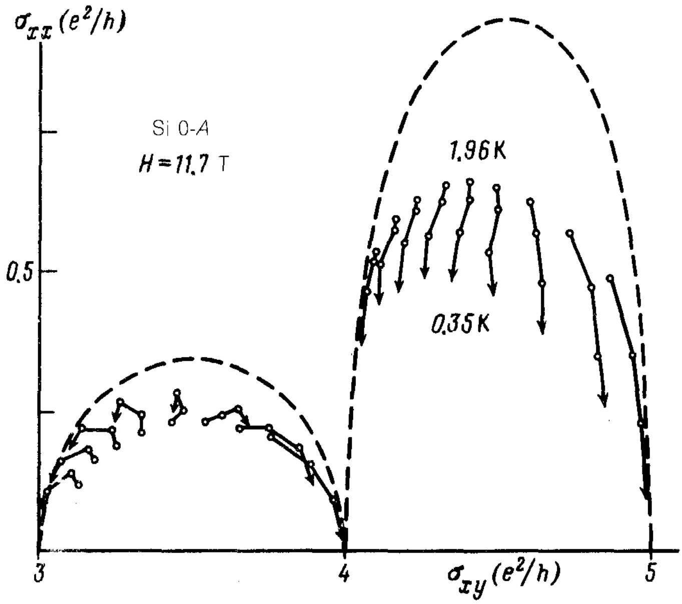

FIG. 2. Phase paths in the $\sigma_{xx} - \sigma_{xy}$ plane for sample Si 0-A at $H = 11.7$ T in the temperature interval 1.96–0.35 K.

Figure 2 shows phase paths for the 0↓ − and 1↑ + sublevels; these paths are typical of all of the sublevels of respectively the zeroth and first Landau levels. It follows from the theory of Refs. 4 and 5 that σ_{xy} varies along the paths (the arrows show the direction of increasing L_ϕ) toward points of integer quantization. A decrease in σ_{xx} at $T<0.6$ K for $N=0$ and at $T<2$ K for $N=1$ occurs for all initial values of σ_{xy}, including values which are approximately half-integers; this decrease therefore cannot be attributed to a simple decrease in the overlap of sublevels. This result is evidence against the conclusion of the theory of Ref. 11 that the results for $\sigma_{xx}(L_\phi)$ and $\sigma_{xy}(L_\phi)$ for various ν conform to a common curve and that as L_ϕ is varied, these results simply shift along this curve. The result of the present paper instead supports the conclusion of Refs. 4 and 5 that there is a set of phase paths in the $(\sigma_{xx}, \sigma_{xy})$ plane. At large values of σ_{xx} the paths run nearly parallel to the ordinate axis, in agreement with Ref. 4.

We have not been able to determine the value of $\sigma^0 = \lim_{T\to 0}\sigma_{xx}^{\max}$ or even to determine whether it is the same for $N=0$ and $N=1$, since this determination would require lower temperatures. All that we can say at this point is that the maxima of σ_{xx} in the zeroth and first Landau levels move closer together as T decreases. The difference between the values of $\sigma_{xx}^{\max}$ for $N=0$ and $N=1$ also decreases rapidly with increasing magnetic field and with an increase in the electron mobility μ (Fig. 3). The apparent reasons for this result are a decrease in the overlap of neighboring sublevels in the first Landau level and thus a decrease in the starting values of $\sigma_{xx}^{\max}$, which are larger by a factor of four for the completely unseparated sublevels than for the completely separated sublevels.

In several of the samples, generally those having a relatively low mobility (see also Refs. 9 and 12), we observe an increase in $\sigma_{xx}^{\max}$ in the zeroth Landau level as

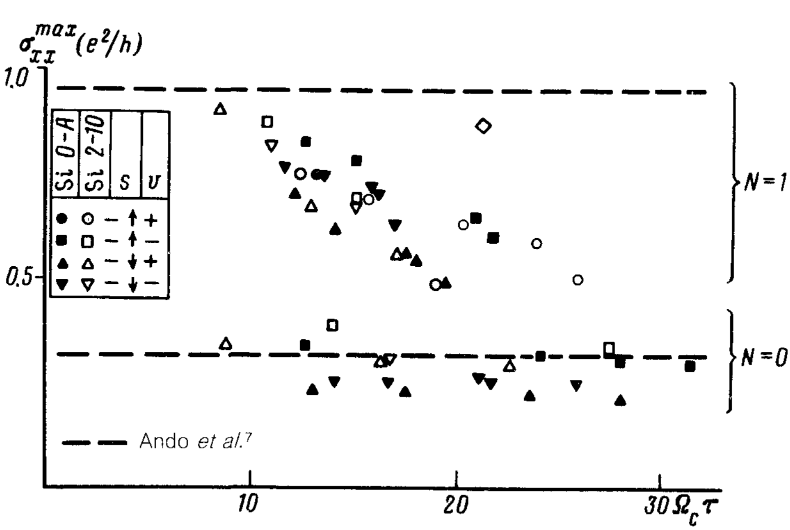

FIG. 3. $\sigma_{xx}^{\max}$ versus the dimensionless parameter $\Omega_c\tau \propto \mu H$ (μ depends on V_g) for two samples at $T=0.35$ K. The rhombus shows the result of Ref. 6 for the 1↑ + sublevel, and at $T=1.5$ K.

$T \to 0$, at least down to $T = 0.35$ K. It may be that the difference in the behavior of $\sigma_{xx}(N=0)$ in the different samples means that σ^0 is not universal and can be either larger or smaller than $\sigma_{xx}^{max}(N=0)$ at the starting point. For the first Landau level, in contrast, the value of σ_{xx}^{max} at the starting point is significantly larger[7] than any estimates of σ^0, so $\sigma_{xx}^{max}(N=1)$ decreases at $T \to 0$ in all of the samples studied.

In summary, it has been shown that at sufficiently low temperatures in a strong magnetic field the magnetoconductivity of silicon MIS structures is not described by a kinetic equation. As the temperature is lowered, quantum-mechanical effects come into play, causing the maxima of the diagonal conductivity for the zeroth and first Landau levels to move closer together. The results can be explained on the basis of a two-parameter scaling theory.

[1)]This condition does not always hold.[8] Since the reason for the difference in the conductivities in the different sublevels of a single Landau level is not clear, we studied only samples with approximately equal values of σ_{xx}^{max} for each N, in order to simplify the interpretation of the results.

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