Excitation of collective oscillations in layered superconducting structures

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A fluxon lattice forms in layered superconducting structures in weak magnetic fields. The absorption of microwave power in the system was found to be resonant in nature and to be related to the excitation of the "magnetic sound" in the fluxon lattice. The theoretical results are in good agreement with the results of recent experiments with high-T_c superconducting single crystals.

Interest in layered superconductors has been revived in recent years. This interest stems from the fact that some high-T_c superconducting compounds are layered superconductors with a Josephson junction between the layers (Bi–Sr–Ca–Cu–O, for example).[1] Furthermore, there is evidence that the order parameter is suppressed on the twinning planes and that a Josephson junction appears on them.[2] A crystal with a regular sequence of such planes can therefore be regarded as a $\{S,N\}$ system or a $\{S,I\}$

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system, i.e., a periodic sequence of superconducting (S) and normal (N) layers or insulating (I) layers. The $\{S,N\}$ or $\{S,I\}$ systems can now be synthesized artificially.[3,4]

If such a system ($\{S,I\}$, for example) is placed in a magnetic field H_e, then at $H_e > H_{J1}$ it will be penetrated by Josephson vortices (fluxons) and the sample will form a fluxon lattice (here H_{J1} is the critical field for the penetration of fluxons). It was shown in Ref. 6 that at frequencies $\omega > (Rc)^{-1}$ slightly damped acoustic oscillations can propagate in a fluxon lattice. We will consider here the effect of an alternating signal on the fluxon lattice and we will show that at frequencies $\omega_n = k_n v_1$ a resonant absorption caused by the excitation of "magnetic sound" occurs in the lattice (here $k_n = 2\pi n/L_z$, v_1 is the velocity of sound that propagates across the layers, and L_z is the thickness of the sample).

Let us consider a $\{S,I\}$ system in a field H_e which is parallel to the S layers. The fluxon lattice and its dynamics are described by the system of equations[6]

$$H'_n = \frac{1}{2\lambda_J^2}\left[\sin\varphi_n + \dot{\varphi}_n\tau + \ddot{\varphi}_n\omega_0^{-2}\right] \tag{1}$$

$$H_n = \frac{1}{2}\sum_m \varphi'_m \exp\left(-a\,|n-m|\right]. \tag{2}$$

Here all the quantities (except time) are dimensionless; the London penetration depth λ is the unit of length, $\Phi_0/(2\pi\lambda^2)$ is the unit of the magnetic field, i.e., the unit which coincides in order of magnitude with H_{c1}; H_n and ϕ_n are the field and the phase difference in the nth I layer, a is the thickness of the S layers, $\lambda_J = [\Phi_0 c/(16\pi^2 j_c\lambda)]^{1/2}$, λ is the dimensionless Josephson length, $\varphi'_n = \partial\varphi_n/\partial x$, $\varphi_n = \partial\varphi_n/\partial t$, $\tau = \hbar/(2eRj_c)$, $\omega_0^2 = 2ej_c/(\hbar C)$; and R, C, and j_c are the resistance, capacitance, and the critical current per unit area of the Josephson junction. The x and y axes lie in the plane of the layers, and the field H_e is directed along the y axis.

At $H_e \gg H_{J1}$ the fluxon lattice is a close-packed lattice. The phases φ_n can then be described as[6]

$$\varphi_n = \mathcal{H}x + \psi_n + \theta_n, \tag{3}$$

where the constant $\mathcal{H}$ is related to the induction B and to the field H_e by the relation

$$\mathcal{H} = Ba = 2H_e \tanh(a/2)[1-f]. \tag{4}$$

The function $f = [32\lambda_J^4 H_e^4 \tanh^4(a/2)]^{-1}$ is assumed to be small in the interval of fields we are considering (see the discussion below). The function ψ_n, whose amplitude is small $[\psi_n \sim (\lambda_J^2\mathcal{H}^2 \sinh a)^{-1} \sin(\mathcal{H}x + \theta_n) \ll 1]$, oscillates spatially with a period of $2\pi\mathcal{H}^{-1}$ along the x axis. The function θ_n describes the structure and the deformation of the fluxon lattice. In the standard case we have $\theta_n^{(0)} = \pi n$ (a triangular lattice). For a typical deformed lattice the equation for θ_n can be obtained by substituting (3) in (1) and (2), expanding $\sin\varphi_n$ in ψ_n, and averaging over the lattice period[6]

$$\lambda_J^2\sum_m \theta''_m \exp[-a|n-m|] = -l_H^2 [\sin(\theta_{n+1}-\theta_n) + \sin(\theta_{n-1}-\theta_n)] + \dot\theta_n\tau + \ddot\theta_n\omega_0^{-2}, \qquad (5)$$

where $l_H = (4\lambda_J^2\mathscr{H}^2 \sinh a)^{1/2}$ is an appreciable length. This equation and expression (4) hold if the following conditions are satisfied:

$$(\lambda_J\sqrt{\sinh a})^{-1} \ll \mathcal{H} \ll 1, \qquad \omega/\omega_0 \ll \lambda_J\mathcal{H}\sqrt{\sinh a}. \qquad (6)$$

Equation (5) generally describes the nonlinear distortions of the fluxon lattice. Volkov,[5] for example, obtained the solutions for θ_n in the form of dislocations in the analysis of the $\{SI_1SI_2SI_1...\}$ system. Such solutions, called supersolitons, were obtained by Ustinov[7] for a single Josephson junction whose parameters vary periodically in space. In each case the nonlinear distortion of the fluxon lattice is attributable to the commensurability effect. The velocity of the magnetic sound which propagates along the layers, $v_\parallel = \omega_0\lambda_J\sqrt{\coth(a/2)}$, does not depend on H_e [if condition (6) holds], while the velocity of sound which propagates across the layers, $v_\perp = \omega_0(a/2\lambda_J) \times [2H_e \tanh(a/2)]^{-1} (\sinh a)^{-1/2}$, decreases with increasing H_e (Ref. 6).

Let us now assume that a current $I = I_\omega \sin(\omega t)$, which deforms the fluxon lattice and gives rise to a magnetic field $h(z,t)$ which varies smoothly across the sample, flows through the sample along the x axis. The amplitude I_ω is assumed to be small but large enough to overcome the pinning forces. The equation for $h(z,t)$, found from (2) and (5), is

$$-(a/l_H)^2 \frac{\partial^2 h}{\partial z^2} + \dot h\tau + \ddot h\omega_0^{-2} = 0. \qquad (7)$$

We can thus easily find the solution we are seeking,

$$h(z,t) = h_\omega \operatorname{Im}\left[\frac{\sinh(\kappa z)}{\sinh(\kappa L)}\exp(i\omega t)\right]. \qquad (8)$$

Here $\varkappa^2 = [(\omega/\omega_0)^2 + i\omega\tau](l_H/a)^2$, $L = L_z/2$ (L_z is the thickness of the sample along the z axis), and the amplitude h_ω is linked with the current I_ω: $I_\omega = (c/2\pi)L_y h_\omega$. Linking the current density $j_x(z,t)$ and the induction field E_x with $h(z,t)$ by means of Maxwell's equations, we can find the absorbed power per unit volume,

$$P = \frac{1}{L_z}\int_{-L}^{L} dz\,\overline{(j_x E_x)} = \frac{1}{4\pi L_z}\,\frac{h_\omega^2\tau\omega_0^2}{\sin^2(\kappa_0 L) + (\kappa_1 L)^2}. \qquad (9)$$

Here the bar means that the average is taken over the time; we have represented $\varkappa$ as $\varkappa = \varkappa_0 + i\varkappa_1$ and assumed $\varkappa_1 \ll \varkappa_0$ [i.e., $\omega \gg (RC)^{-1}$] and $\varkappa_1 L \ll 1$. It follows from (9) that the absorptive power has several sharp peaks when

$$\kappa_0 = 2(\omega/\omega_0)\lambda_J\mathcal{H}\sqrt{\sinh a}/a = 2\pi n/L_z, \qquad n = 1, 2, \ldots \qquad (10)$$

The amplitude of the peaks in the field interval (6) under consideration depends only slightly on H_e, and the peaks are spaced uniformly with respect to H_e. At $a \ll 1$ the spacing between the peaks is $\Delta H_e = (\pi\omega_0/\omega)[\lambda_J L_z \sqrt{a}]^{-1}$, i.e., it increases with decreasing L_z. At $L_z = 0.1$ mm and $\lambda = 10$ μm ($j_c \approx 10^3$ A/cm^2) the value of ΔH_e is estimated to be $\approx 10^{-3}$ $\omega_0/(\omega\sqrt{a})$ Oe. The estimated value of ΔH_e and that of the H_e and L_z dependence of the resonance are in agreement with the results of Refs. 2 and 8, in which similar resonances in Y–Ba–Cu–O single crystals were observed. We can thus assume that a resonant excitation of the magnetic sound in the fluxon lattice was observed in the experiments and that the Josephson junctions are formed by the regularly spaced twinning planes. The experiments were carried out in weak fields. Note that the fluxon lattice can form in very low fields, since $H_{j1} \approx \lambda_J^{-1} \ll 1$ when $a > 1$ and $H_{J1} \approx \sqrt{a/\lambda_J} \ll 1$ when $a \ll 1$.

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