Possibility of Bose condensation of magnons excited by incoherent pump

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A new type of instability of spin waves with an incoherent external pump is predicted. This instability would result in a sharp increase in the number of magnons at the bottom of the spectral band.

The application of varying coherent magnetic fields to a system of spin waves in a magnetic material is known to drive various types of parametric instabilities. Highly nonequilibrium states arise. Such states are usually described by a dynamic theory.[1] In the present letter we show that during incoherent pumping there can be a Bose condensation of quasiequilibrium magnons even below the threshold for a parametric instability (the threshold rises markedly under these conditions).

We start from a kinetic equation for the magnon occupation numbers $n_{\mathbf{k}}$:

$$\frac{d}{dt} n_{\mathbf{k}} = I^{(4)}\{n_{\mathbf{k}}\} + f_{\mathbf{k}} + r_{\mathbf{k}}, \qquad (1)$$

where $I^{(4)}\{n_{\mathbf{k}}\}$ is a four-magnon collision integral (we are not considering three-particle collisions), $f_{\mathbf{k}}$ is an incoming term which describes the probability for the creation of a magnon by the external agent, and $r_{\mathbf{k}}$ is a relaxation term which is responsible for the interaction of the magnons with the heat reservoir (the magnons of other branches and phonons). We seek a solution of Eq. (1) in the form

$$n_{\mathbf{k}} = \{\exp[(\epsilon_{\mathbf{k}} - \mu)/T] - 1\}^{-1}, \qquad (2)$$

where $\epsilon_{\mathbf{k}}$ is the magnon energy ($\hbar = k_B = 1$). The collision integral then vanishes, and the state of this system is characterized by two parameters: the chemical potential μ and the temperature T, as functions of the intensity of the external pump source. It is not difficult to see that an increase in $n_{\mathbf{k}}$ leads to an increase in T and/or μ. There is a singularity along the μ scale, at the point at which the chemical potential touches the bottom of the magnon band: $\mu = \min(\epsilon_{\mathbf{k}})$. At this point, the occupation number becomes infinite. A thermodynamic description of the spin-wave system on the basis of Eqs. (1) and (2) is then no longer valid. To some extent the situation is reminiscent of equilibrium Bose condensation. A spin-wave system with a condensate can be analyzed by a method similar to that proposed in Ref. 2. Incorporating the magnon-magnon interactions in the approximation of a self-consistent field shows that anomalous binary correlations arise for the operators which create and annihilate quasiparticles. We will restrict the analysis below to the subcritical region.

Introducing the notation

$$N \equiv V\int d^3 k n_{\mathbf{k}}, \quad F_N \equiv V\int d^3 k f_{\mathbf{k}}, \quad R_N \equiv V\int d^3 k r_{\mathbf{k}},$$
$$E \equiv V\int d^3 k \epsilon_{\mathbf{k}} n_{\mathbf{k}}, \; F_E \equiv V\int d^3 k \epsilon_{\mathbf{k}} f_{\mathbf{k}}, \; R_E \equiv V\int d^3 k \epsilon_{\mathbf{k}} r_{\mathbf{k}}, \tag{3}$$

we can, without difficulty, derive balance equations for the total number of particles and the energy of the magnon system from (1):

$$\frac{d}{dt}N = F_N + R_N, \quad \frac{d}{dt}E = F_E + R_E.$$

These equations determine the evolution of μ and T. For simplicity, we can write the relaxation terms in the τ approximation:

$$R_N = \tau_N^{-1}[N - N^{(0)}], \quad R_E = \tau_E^{-1}[E - E^{(0)}],$$

where $N^{(0)}$ and $E^{(0)}$ are found from (3) with $\mu = 0$. Steady-state solutions for T and μ as functions of the pump intensity F are found from the integral equations

$$\tau_N F_N = N - N^{(0)}, \quad \tau_E F_E = E - E^{(0)}. \tag{4}$$

A numerical analysis of these equations for spin waves with a spectrum $\epsilon_{\mathbf{k}} = (\epsilon_0^2 + s^2 k^2)^{1/2}$ (characteristic of antiferromagnets) shows that $T(F)$ and $\mu(F)$ are monotonically increasing functions (with increasing F).

In the case $\tau_E \ll \tau_N$, the change in the temperature of the system below the instability threshold can be ignored. The first equation in (4) is then sufficient for calculating the change in the chemical potential. This inequality holds in magnetic materials (the ferrite YIG, the antiferromagnet $FeBO_3$, etc.) in which the predominant processes by which the spin waves interact with the reservoir are processes which conserve the total number of magnons of the spectral branch of interest. One such process is the decay of a magnon into a magnon and a phonon.

In an antiferromagnet excited by an incoherent pump which has a frequency band of Lorentzian shape with a width $\Delta\Omega$ and a maximum at the frequency Ω, at which the amplitude of the varying magnetic field is h, the incoming term is

$$f_{\mathbf{k}} = 2|h V_{\mathbf{k}}|^2 (2n_{\mathbf{k}} + 1)\Delta\Omega[(2\epsilon_{\mathbf{k}} - \Omega)^2 + \Delta\Omega^2]^{-1}, \tag{5}$$

where $V_{\mathbf{k}} = g^2(H + H_D/2)\epsilon_{\mathbf{k}}^{-1}$ is the coupling coefficient describing the coupling with the pump field, H is a static magnetic field, H_D is the Dzyaloshinskiĭ field, and g is the gyromagnetic ratio. Using (5), and working from the condition $\mu = \min(\epsilon_{\mathbf{k}})$, we find an expression for the instability threshold h_*

$$h_*^2 = T^4\,\Phi\left(\frac{\epsilon_0}{T}, \frac{\Omega}{T}, \frac{\Delta\Omega}{T}\right)/g^4(2H + H_D)^2 \Delta\Omega \tau_N,$$

where

$$\Phi(x, y, z) = 2\cdot [I_1(x, x) - I_1(0, x)]/I_2(x, y, z),$$

$$I_1(x, y) \equiv \int_{y-x}^{\infty} dt \, \frac{(t+x)[(t+x)^2 - y^2]^{1/2}}{e^t - 1},$$

$$I_2(x, y, z) \equiv \int_{x}^{\infty} dt \, \frac{(t^2 - x^2)^{1/2}}{t[(2t-y)^2 + z^2]} \operatorname{cth}[(t-x)/2].$$

For comparison, the threshold for the parametric instability driven by the same incoherent pump is given by[1] $(h_c V_{\mathbf{k}})^2 \approx \gamma_{\mathbf{k}} \Delta\Omega$, where $\gamma_{\mathbf{k}}$ is the relaxation rate of magnons with $\epsilon_{\mathbf{k}} \approx \Omega$. Of primary interest is the opposite behavior of h_* and h_c as functions of $\Delta\Omega$. Estimates of h_* and h_c for $FeBO_3$ ($H_D \approx 100$ kOe) with $T = 1$ K, $\epsilon_0 = 20$ GHz, $\Omega = 40$ GHz, $\Delta\Omega = 2$ GHz,$\tau_N^{-1} = 1$ kHz, and $\gamma_{\mathbf{k}} = 1$ mHz yield $h_* \sim 10^{-1}$ Oe and $h_c \sim 30$ Oe.

In summary, Bose condensation of magnons can be observed at liquid-helium temperatures. The formation of a condensate implies the onset of an oscillating macroscopic magnetic moment, which can be detected from the coherent electromagnetic emission with a frequency which is a multiple of the size of the spectral gap. A similar effect may arise above the threshold for a parametric resonance of magnons.

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[2]E. M. Lifshitz and L. P. Pitaevskiĭ, *Statistical Physics*, Pergamon, Oxford, 1980, Ch. 3.

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