Relaxation of unstable magnetization precession in ^{3}He-A

I. A. Fomin and D. V. Shopova
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

(Submitted 3 July 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **50**, No. 3, 143–145 (10 August 1989)

During pulsed NMR in ^{3}He-A, the magnetization relaxation occurs through a propagation of fronts away from cell walls oriented perpendicular with respect to the magnetic field. The front propagation velocity is found in the limit of intense diffusion.

The spatially uniform precession of the magnetization in the superfluid A phase of ^{3}He is unstable.[1] Specially designed pulsed NMR experiments have shown that this instability determines the lifetime of the induction signal if the initial angular deviation of the magnetization is not too small.[2,3] The existing linear theory is successful in describing only the beginning of the evolution of the instability.[4] Our purpose in the present study was to derive a theory to describe the magnetization relaxation in ^{3}He-A under conditions of pulsed NMR in the nonlinear region, i.e., in a situation in which the deviation of the precession from uniform can no longer be treated as small. This theory is based on the results of Kolmogorov, Petrovskiĭ and Piskunov[5] and their development by Kamenskiĭ and Manakov.[6] It was shown in Ref. 5 for the nonlinear diffusion equation and in Ref. 6 for a wider class of problems that in a situation in which the perturbations which induce an instability are one-dimensional and localized, the transition to the equilibrium state is of a combustion nature; i.e., there is a front which propagates at a constant velocity. On one side of this front is the unstable initial state, and on the other side is an equilibrium state. The velocity at which this front moves can be found from equations linearized around the initial state.

To determine whether a transition to an equilibrium state can occur in this fashion in the problem with which we are concerned here, it is necessary, according to Kamenskiĭ and Manakov, to study the asymptotic behavior of the solutions of the linearized problem at a large distance (z) from the initial perturbation and after a long time t under the condition $z = Vt$, where $V =$ const. This asymptotic behavior is determined by the saddle points $k_s(V)$ of the Fourier representation of the solution. The points k_s are found as the roots of the equation

$$\frac{d}{dk}(ikV + \Gamma(k)) = 0, \qquad (1)$$

where $\Gamma(k)$ is the growth rate of perturbations with wave vector k. A relaxation can occur in the manner described above if there exists a value $V = V_c$ such that for all $V > V_c$ the real part of the expression $ik_s V + \Gamma(k_s)$ is negative. The minimum value of V_c is the front propagation velocity. This condition holds for the growth rate of the instability of the precession in ^{3}He-A, but the expression for $\Gamma(k)$ for arbitrary values of the parameters in it is quite complicated,[4] and the velocity V_c cannot be found analytically. The analysis simplifies in the case of intense diffusion, i.e., when the parameter $\Lambda \equiv 2D\omega_L/c^2$ is large. Here ω_L is the plasma frequency, and D and c^2 are components of the spin-diffusion tensor and the square-velocity tensor for spin waves, respectively, which are important for the particular geometry of the problem. The parameter Λ depends on the temperature; in the limit $T \to T_c$ we have $\Lambda \to \infty$, while far from T_c we would have $\Lambda \lesssim 1$ under typical conditions. We will restrict the discussion here to the case of large Λ; the results of a numerical analysis for $\Lambda \sim 1$ will be reported in a separate paper. Retaining in the expression for $\Gamma(k)$ the terms which are of leading order in $1/\Lambda$, we find

$$\Gamma(k) = \frac{\Omega}{4\omega_L} \sin\beta \left(3\frac{3-\cos\beta}{1+\cos\beta}\right)^{1/2} ck - Dk^2. \qquad (2)$$

After going through the necessary calculations, we can show that a limiting velocity exists and is given by

$$V_c = V_{c\infty} = c\frac{\Omega}{4\omega_L} \sin\beta \left(3\frac{3-\cos\beta}{1+\cos\beta}\right)^{1/2}. \qquad (3)$$

This is the front velocity in our problem. In expressions (2) and (3), β is the initial magnetization deviation angle, and Ω is the frequency of longitudinal oscillations.

Under what conditions will the initial perturbations satisfy the requirements of Ref. 6? It was shown in Ref. 3 that the chamber walls constitute an important source of perturbations and that the orientation of the walls with respect to the magnetic field $\mathbf{H}_0$ is important. The boundary conditions require that the vector $\mathbf{l}$, which characterizes the orientation of the orbital part of the order parameter in ^{3}He-A, be directed along the normal to the wall. Far from the boundaries, at equilibrium, we would have $\mathbf{l} \perp \mathbf{H}_0$. If $\mathbf{H}_0$ lies in the plane of the wall, we can satisfy both requirements. If, on the other hand, the field $\mathbf{H}_0$ runs perpendicular to the boundary, then these requirements are incompatible, and a transition layer with a thickness on the order of the dipoles length $l_D \sim 10^{-3}$ cm appears. In this layer, the orientation of $\mathbf{l}$ changes from $\mathbf{l} \perp \mathbf{H}_0$ far from the boundary to $\mathbf{l} \parallel \mathbf{H}_0$ right at the boundary. We will assume that the volume of helium of interset is bounded exclusively by walls oriented parallel to $\mathbf{H}_0$ (side walls) or perpendicular to $\mathbf{H}_0$ (bases). The distance between the side walls is appreciable in comparison with the distance between the bases. The spin precession frequency in ^{3}He-A depends on the relative orientation of $\mathbf{l}$ and $\mathbf{H}_0$, so the local precession frequency near the bases differs from the spin precession frequency in the interior. As a result, a state whose spatial uniformity is disrupted near the bases arises *after* the deflecting pulse has ended. This nonuniformity serves as an initial perturbation. All of the conditions for the applicability of the approach of Ref. 6 turn out to be satisfied, and it can be asserted that in this geometry a front travels away from each base of the cell after the deflecting pulse has ended. The front velocity is given by expression (3), in which c is c_l: the larger of the two velocities which appear in the gradient energy of ^{3}He-A. The complete relaxation time τ is thus proportional to the distance between the basis, L, and is given by $\tau = L/2V_c$. We would expect that the picture of the relaxation would not be qualitatively different in a more complex geometry, but the fronts could not be planar, because of both the more complex shape of the initial perturbations and the anisotropy of c^2. The front width λ can be estimated from the energy dissipation rate: $\lambda \sim D / V_c \sim \Lambda^c /_{\Omega} \sim \Lambda l_D$. From the equations of motion we can also find asymptotic expressions for the solutions on the two sides of the front. In contrast with the examples discussed in Ref. 6, the two asymptotic expressions are not functions of the combination $z - V_c t$ exclusively. This circumstance apparently indicates that the shape of the front varies in time.

The picture of the relaxation drawn here agrees qualitatively with the experimental results of Refs. 2 and 3. Unfortunately, a quantitative interpretation of these results is ruled out because the cell geometry in the experiments of Refs. 2 and 3 did not meet the requirements formulated above. It would be useful to carry out some similar experiments with a suitably altered cell geometry, with provision for measurements of the front velocity.

Since instability similar to that which we have been discussing here occurs in the antiferromagnetic phase of solid ^{3}He (Ref. 7), the magnetic relaxation in this phase may also occur in the manner described above.

We wish to thank V. G. Kamenskiĭ and S. V. Manakov for useful discussions.

[1] I. A. Fomin, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 179 (1979) [JETP Lett. **30**, 164 (1979)].
[2] A. S. Borovik-Romanov *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 390 (1984) [JETP Lett. **39**, 469 (1984)].
[3] Yu. M. Bun'kov *et al.*, Zh. Eksp. Teor. Fiz. **88**, 1218 (1985) [Sov. Phys. JETP **61**, 719 (1985)].
[4] I. A. Fomin, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 387 (1984) [JETP Lett. **39**, 466 (1984)].
[5] A. N. Kolmogorov *et al.*, Byull. MGU. Mat. Mekh. **1**, 1 (1937).
[6] V. G. Kamenskiĭ and S. V. Manakov, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 499 (1987) [JETP Lett. **45**, 638 (1987)].
[7] T. Ohmi *et al.*, Prog. Theor. Phys. **73**, 1075 (1985).

Translated by Dave Parsons