Conductivity of a quasi-one-dimensional channel in the ballistic regime

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The degree to which the ballistic conductivity of a quasi-one-dimensional channel depends on the actual shape of $V(x)$ of the external potential which forms the given channel is discussed. The total number l_* of the electronic subbands, which are situated below the Fermi level in the given channel, is shown to be sensitive to the actual profile of $V(x)$.

The conductivity σ of quasi-one-dimensional electronic channels, which has recently been studied extensively (see, in particular, Refs. 1–3), is usually calculated using a square-shaped external potential $V(x)$ which forms the conducting channel. Such an approximation is reasonable if the particular features of the dependence $V(x)$ are not important in explaining the observable characteristics of the conductivity. Otherwise, a more suitable $V(x)$ model should be used to determine the properties of the conducting channel with variable parameters.

Our purpose in the present letter is to determine the degree of "sensitivity" of the various characteristics of the conductivity of the channel to the $V(x)$ model.

1. The most curious observable characteristic of the conductivity σ is its abrupt increase with the variation of the potential difference V_g between the gate and the electronic channel. The magnitude of the individual jump, $\Delta\sigma = e^2/h$, in this case does not depend on the number of the step.[1] Calculation of σ for the electronic spectrum with an arbitrary discrete part of ε_l in the ballistic regime shows that the conductivity does have steps ($\Delta\sigma$) which are attributable to the appearance of new conduction subbands in the channel due to a change in V_g:

$$\sigma_y \equiv \sigma = \frac{e^2}{h} \sum_l [1 + \exp(\frac{\epsilon_l - \mu}{T})]^{-1}, \quad \Delta\sigma = e^2/h \ . \qquad (1)$$

Here μ is the chemical potential, and T is the temperature. The conductivity σ such as that in (1) was determined for a square channel in Refs. 1 and 3. Result (1) obviously does not depend on the explicit behavior of $V(x)$.

2. The next point, which has so far not been discussed, has to do with the fact that the number of steps on the $\sigma(V_g)$ curve increases approximately linearly with an increase in $V_g - V_g^m$, where V_g^m is the maximum value of V_g at which the channel still exhibits conducting properties.[1] This number of steps can be associated with the maximum number l^* of the conduction subbands in the channel which are filled with electrons. It thus follows from Ref. 1 that

$$l^*_{\exp} \propto (V_g - V_g^m).$$ (2)

The value of l^* for an arbitrary potential $V(x)$ cannot be calculated in the general form, as has been done for the conductivity σ. In the two limiting cases, however—the square and the parabolic potential—the final results for l^* in the quasiclassical approximation have the same structure

$$l^* = c(N_L a)^{1/2} \qquad l^* \gg 1,$$ (3)

where c is the numerical factor on the order of unity, N_L is the number of electrons per unit length of the channel, and $2a$ is the channel width.

To determine the realtionship between N_L,a, and the potential $V(x)$, we will write the quasiclassical condition for equilibrium in the channel which is situated sufficiently far (a distance $|b-c| \gg a$) from the metallic electrodes (Fig. 1)

$$V(x) + e\varphi(x) = 0, \quad \varphi(x) = \frac{e}{\kappa}\int_{-a}^{+a} n(s)\ln\frac{L}{|x-s|}ds, \quad n(x)|_{\pm a} = 0$$ (4)

$$\int_{-a}^{+a} n(s)ds = N_L, \quad -a \leqslant x \leqslant +a.$$

Here $\varkappa$ is the dielectric constant of the substrate, $n(x)$ is the electron density in the channel, and L is the channel length in the y direction. Relation (4) holds if $a \gg a_b$, where $a_b = \varkappa\hbar^2/(m^* e^2)$, and m^* is the effective mass.

Making use of the Chebyshev polynomials, we obtain from (4) two general solutions

$$\frac{1}{\pi}\int_{-a}^{+a}\frac{V(s)ds}{(a^2-s^2)^{1/2}} + \frac{e^2 N_L}{\kappa}\ln\frac{2L}{a} = 0,$$ (5)

$$\frac{1}{\pi}\int_{-a}^{+a}\frac{dV}{ds}\frac{sds}{(a^2-s^2)^{1/2}} = N_L.$$ (6)

Relation (5) depends only slightly on the explicit form of $V(x)$, and relation (6) is highly sensitive to the behavior of $V(x)$ in the channel. In particular, for a square potential, when dV/dx has the shape of δ-functions at the ends of the interval

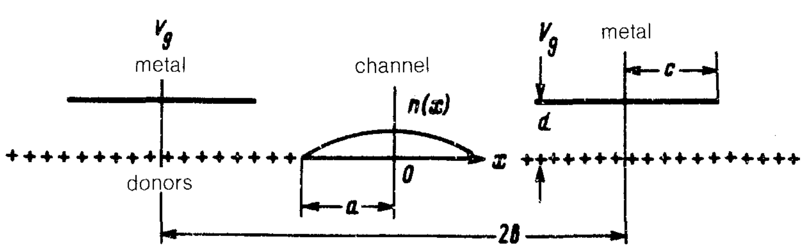

FIG. 1.

$x = \pm a$, relation (6) becomes meaningless because its left side is divergent. The theory which makes use of the square confining potential thus encounters qualitative difficulties stemming from the need to remove singularities from relation (6). A smooth potential $V(x)$ removes any doubt regarding this divergence, but, at the same time deprives the theory of the advantages of a rectangular approximation for $V(x)$, which is frequently used, for example, to calculate the conductivity of the channel which has a slight nonuniformity in the direction in which the current flows (see Ref. 3).

Let us now consider an actual model which would allow us to obtain quasi-one-dimensional channels and which was numerically simulated by Laux *et al.*[4] in the region of small values N_L (Fig. 1). Assuming that the channel width a is small compared with the spacing between the adjacent metallic strips (but large compared with a_b)

$$|b-c| \gg a \gg a_b, \quad d \ll a, b, c \quad , \tag{7}$$

and solving the appropriate electrostatic problem which corresponds to the geometry of the figure, we can easily see that the confining potential in the channel is primarily a parabolic potential

$$V(x) \approx V_0 + \frac{1}{2} kx^2, \; x \lessgtr a \tag{8}$$

$$k = \frac{e}{2\kappa}(2bn_s^+ - N_L)\frac{1}{b^2-c^2}\left(\frac{c}{b} + \frac{b}{\sqrt{b^2-c^2}}\tan^{-1}\frac{(b^2-c^2)^{1/2}}{c}\right) \tag{9}$$

$$eN_L \int_{-c}^{+c} \frac{dx}{\sqrt{c^2-x^2}} \ln\frac{|b^2-x^2|}{a^2} = 2\pi\kappa(V_g - V_g^m) \tag{10}$$

$$a^2 = 2e^2N_L/(k\kappa), \quad n(x) = \frac{2N_L}{\pi a}\left(1 - \frac{x^2}{a^2}\right)^{1/2}. \tag{11}$$

Here n_s^+ is the two-dimensional density of the donors which account for the filling of the $2D$ electronic system. For simplicity, we ignored the spatial separation of donors and $2D$ electrons; $n(x)$ is the electron density in the channel for $-a \leqslant x \leqslant +a$.

Relations (9)–(11) determine k, N_L, and a as functions of $V_g - V_g^m$ and the geometry of the problem. It is clear, in particular, that N_L depends approximately linearly on $V_g - V_g^m$, in good agreement with the numerical calculations of Ref. 4.

Using the Thomas–Fermi approximation, which holds in our case in the region $a \gg a_b$, we can write the effective potential $\tilde{V}(x)$, which quantizes the motion of electrons in the x direction, in the following form[6] [$\varphi(x)$ was taken from (4) and $V(x)$ was taken from (8)]:

$$\tilde{V}(x) = V(x) + e\varphi(x) \approx -\frac{\pi\hbar^2}{m^*} n(x). \tag{12}$$

For $n(x)$, the classical definition of $n(x)$ in (11) is legitimate to the extent that the condition $a \gg a_b$ is satisfied. Making use of the quasiclassical potential $\tilde{V}(x)$ [Eq. (12)], it is not difficult to calculate the total number of levels in this well,[6] from which we find l^* in (3).

Taking into account a, N_L, and k in (9)–(11) and the expression for l^* in (3), we can easily determine the dependence of l^*_{theor} on $V_g - V_g^m$:

$$l^*_{\text{theor}} \propto (V_g - V_g^m)^\lambda, \quad \lambda \gtrsim 3/4 \quad , \tag{13}$$

Here $\gtrsim$ in the evaluation of λ takes into account the dependence of the curvature k in (9) on $V_g - V_g^m$. Result (13) is a good approximation of the experimental behavior of l^* in (2), although it is not exactly equal to it. It is quite conceivable that the behavior of l^*_{exp} in (2) depends on the nonuniformity of the channel in the direction in which the current flows.

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