Excitation of ultrarelativistic plasma waves by pulse of electromagnetic radiation

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The excitation of plasma waves by a relativistically intense pulse of electromagnetic radiation is shown to depend strongly on the rise time of the pulse. The characteristics of the excited wave are determined. The evolution of the pulse itself is studied. The possible acceleration of particles by the field of the longitudinal wave which is excited is discussed.

The increased interest in the excitation of fast plasma waves in a plasma is a result of the present effort to develop laser methods for accelerating particles, which would be characterized by an exceedingly high acceleration rate.[1] The most experimental and theoretical progress is being achieved in the development of beat-wave accelerators,[2] with two-frequency laser pulses. Because of several difficulties associated with the resonant nature of the excitation of fast plasma waves in a beat-wave accelerator, and also because of rapid developments in the field of ultrashort laser pulses,[3] another approach is also attracting interest. This is a nonresonant method of exciting fast plasma waves by short laser pulses; it was the method originally proposed in Ref. 4. In the present letter we examine this excitation method in the case of a relativistically intense electromagnetic pulse $(eE_\perp/m\omega_0 c \gg 1)$.

A linearly polarized electromagnetic pulse with a carrier frequency $\omega_0 \times (\omega_0 \gg \omega_p = \sqrt{4\pi e^2 N_0/m_e}$, where N_0 is the density of the immobile ions) is propagating along the x axis. It creates perturbations of the plasma-electron density and some longitudinal fields, which are described by the dimensionless electrostatic potential $\psi = e\varphi/mc^2$. We write the dimensionless vector potential $A = p_\perp/mc$ which characterizes the electromagnetic pulse in the form $A = \frac{1}{2}[a(\xi,t)\exp(-i\omega_0 t + ik_0 x) + \text{c.c.}]$, where k_0 is the wave vector of the radiation, which is related to ω_0 by the linear dispersion relation $(\omega_0^2 = k_0^2 c^2 - \omega_p^2)$. In addition, $a(\xi,t)$ is a complex amplitude, which is a function of the variables t and $\xi = x - v_g t$ $(v_g = c^2 k_0/\omega_0)$. Assuming that the variations in $\psi(\xi,t)$ and $a(\xi,t)$ in time are slow $(\partial/\partial t \ll c\partial/\partial\xi)$, and assuming that the plasma density is fairly low $[\omega_0^2/\omega_p^2 \gg (1 + a^2/2)(1+\psi)^{-2}]$, we find a system of coupled equations for a and for the low-frequency component of the potential, ψ_0, from the relativistic hydrodynamic equations for cold electrons and from Maxwell's equations:

$$2i\omega_0 \frac{\partial a}{\partial t} + \frac{\omega_p^2}{\omega_0^2} c^2 \frac{\partial^2 a}{\partial \xi^2} + 2v_g \frac{\partial^2 a}{\partial\xi\partial t} = -\omega_p^2 \frac{\psi_0}{1+\psi_0} a \qquad (1)$$

$$\frac{d^2\psi_0}{d\xi^2} - k_p^2 \frac{1 + |a|^2/2 - (1+\psi_0)^2}{2(1+\psi_0)^2} = 0, \qquad (2)$$

where $k_p = \omega_p/v_g$.

By virtue of this inequality, oscillations of the longitudinal potential at harmonics of the carrier frequency can be ignored. Equations similar to Eqs. (1) and (2) were studied in Ref. 5 in the slightly relativistic case. With $a = 0$, Eq. (2) describes free plasma oscillations.[6] If the pulse is circularly polarized, the amplitude of the rf field in (2) should be replaced by $a\sqrt{2}$.

Integrating Eq. (2) for a given electromagnetic pulse ($\partial a/\partial t = 0$) of square shape ($|a|^2 = \text{const}$ at $-L<\xi<0$, $|a|^2 = 0$ at $\xi < -L, \xi > 0$) in the region occupied by the pulse, we find

$$\xi = 2\sqrt{1+|a|^2/2}\,E(R,k) - 2\sqrt{(|a|^2/2-\psi_0)\psi_0/(1+\psi_0)}\,, \tag{3}$$

where $E(R,k)$ is the incomplete elliptic integral of the second kind, and $R = \arcsin[\sqrt{(2+|a|^2)\psi_0}/\sqrt{|a|^2(1+\psi_0)}$ and $k = |a|/\sqrt{2+|a|^2}$ are its argument and modulus, respectively.

According to (3), in the region of the ultrarelativistic electromagnetic pulse ($|a| \gg 1$) the potential ψ_0 oscillates between 0 and $|a|^2/2$ with a period $\lambda_\parallel = 2\sqrt{2}|a|k_p^{-1}$ (Fig. 1). An electromagnetic pulse of length $L = L_s = \lambda_\parallel s (s = 1,2,3,...)$ does not leave a plasma wave behind itself. If, on the other hand, the length of the pulse differs from L_s, even by a relatively small amount, $|L - L_s| \gtrsim |a|^{-1}k_p^{-1}$, the amplitude of the relativistic fast plasma wave behind the pulse and the period of this wave will be approximately the same as the corresponding quantities in the pulse region. This fact is associated with the relativistic increase in the mass at $\psi > 1$, which has the consequence that the work performed by the rf pressure force on the plasma electrons at the trailing edge of the pulse (under the condition $\psi_0 = \psi_{02} \gg 1$) is smaller by a factor of $(1 + \psi_{02})$ than that at the leading edge. For the same reason, again in the case in which the trailing edge of the electromagnetic pulse is not sharp the effect of the shape of this edge on the efficiency of the excitation of the fast plasma wave left behind the pulse (also called a "wake field") will be of minor importance.

If the rise time of the pulse is short, $\delta\xi_1 k_p < |a_1|^{-1}$, we can use the equations

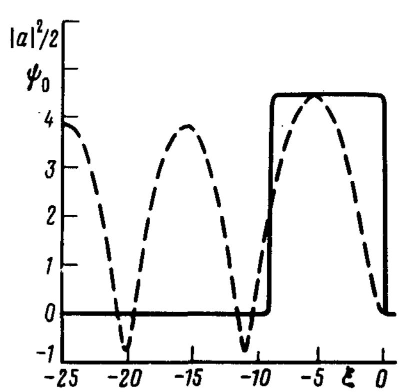

FIG. 1. The quantity $|a|^2/2$ (solid line) and the dimensionless charge-separation potential (dashed line) for the case $a = 3$, $\omega_0/\omega_p = 100$.

derived for a pulse with a sharp front, and the amplitude of the wakefield fast plasma wave will be at the maximum. Under the condition $\delta\xi_1 k_p > |a_1|^{-1}$, an increase in the rise time of the pulse will result in a decrease in the amplitude of the fast plasma wave which is excited, to a value $|a_2|/\sqrt{2}$ (for an electromagnetic pulse with a sharp trailing edge). Finally, if the amplitude rises and decays slowly at the leading and trailing edges of the pulse, $\delta\xi_{1,2} k_p > |a_{1,2}|^{1/2}$, no wakefield wave will be excited at all.

This analysis has used the approximation of a given electromagnetic pulse. We turn now to a discussion of the evolution of the pulse during the excitation of fast plasma waves. The excitation of a wakefield fast plasma wave results in a loss of energy from the pulse at a rate (per unit time)

$$dW_p/dt = {}^1/_2 (\omega_p/\omega_0)^2 \omega_p \int_{-\infty}^{\infty} d\xi\, (\partial |a|^2/\partial\xi) k_p^{-1}\, \psi_0/(1+\psi_0)$$

and a change in the magnitude of the complex amplitude. For the pulse discussed above, with steep edges (i.e., with $d\xi_{1,2} k_p < |a|^{-1}$), the shape of the pulse can be regarded as given as long as the distortions near the leading edge are small: $t < t_{\rm non} = \omega_p^{-1}(\omega_0/\omega_p)^2 |a|^{-1}$. A numerical solution of Eqs. (1) and (2), however, shows that efficient excitation continues over times comparable to or longer than $t_{\rm non}$. This effect is demonstrated by Fig. 2, which shows the same pulse as in Fig. 1, at $t = t_{\rm non}$. Note that the increase in $a \sim eE_\perp/cm\omega(\xi,t)$ near the leading edge of the pulse is accompanied by a decrease in the local frequency $\omega(\xi,t)$; the dotted line in Fig. 2 shows the ratio of this frequency to the initial carrier frequency ω_0. As follows from Eqs. (1) and (2), the total number of quanta, $\sim \int_{-\infty}^{\infty} d\xi (E_\perp^2/\omega(\xi,t))$, is conserved. At $t > t_{\rm non}$, the excitation of the fast plasma wave in fact occurs at a slightly higher rate. When the local value of the frequency at the leading edge is reduced further, however, the calculations on this process can be pursued in this example only to (4–5)$t_{\rm non}$, at which $\omega_{\rm min}(\xi,t)$ becomes comparable to ω_p, and Eqs. (1) and (2) become inapplicable.

A laser pulse with a wavelength $\lambda_0 \sim 10\ \mu$m, an intensity $I \sim 10^{17}$ W/cm^2 ($a \sim 3$), and an edge length of 0.1 ps in a plasma with a density $N_0 \sim 10^{15}$ cm^{-3} ($\omega_0/\omega_p = 100$)

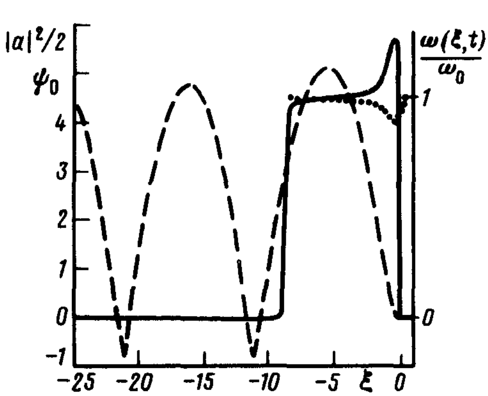

FIG. 2. The same quantities as in Fig. 1, after a time $t = t_{\rm non}$. The dotted line shows the ratio of the local frequency of the electromagnetic radiation, $\omega(\xi, t_{\rm non})$, to ω_0.

will excite fast plasma waves with $E \gtrsim 2$ GeV/m, according to the results derived above. Electrons with an initial energy of 50 MeV could acquire an energy of 1 GeV in the field of the fast plasma wave[4] over a distance of only $x_{\rm non} \approx ct_{\rm non} \sim 50$ cm.

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