Modulational instability of electromagnetic-spin waves in ferrite film

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An instability of spin waves has been observed in a magnetic film at extremely low pump power levels, in a narrow frequency interval near the lower boundary of the magnetostatic-wave spectrum. Retardation must be taken into account in order to interpret this effect. Several parameters of the instability are estimated.

Experimental procedure and results. The spin waves were excited by a length of an asymmetric microwave stripline 0.5 mm wide and 2.5 cm long on a polychore substrate 0.5 mm thick. One end of the line was an open circuit. A monochromatic microwave signal with a frequency $f = \omega/2\pi = 5468$ MHz was applied to the other end. An yttrium iron garnet (YIG) film was applied to an antenna which was placed in a magnetic field $\mathbf{H}_0$, which was directed perpendicular to the plane of the film. We studied the microwave power reflected from the antenna, p_{ref}, and the spectrum of the reflected signal as we varied the strength of the magnetic field, H_0, and the power of the microwave signal fed to the antenna, p_{inc}.

Figure 1 shows a plot of $P_{\text{ref}}(H_0)$ at $P_{\text{inc}} = 10\ \mu$W. These experimental results

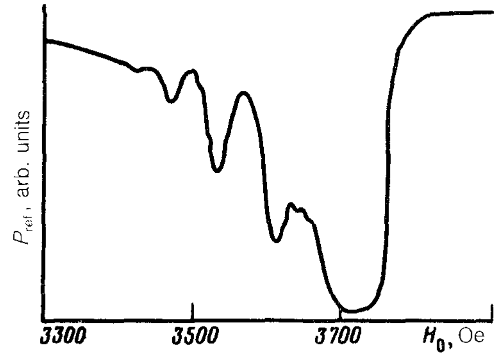

FIG. 1. Reflected power versus the magnetic field.

correspond to an epitaxial YIG film in the (111) orientation with a thickness $S = 11$ μm and a ferromagnetic-resonance linewidth $2\Delta H = 0.38$ Oe. It can be seen from Fig. 1 that in fields $H_0 < H_{01}$ ($H_{01} = \omega/\gamma + 4\pi M$, where γ is the gyromagnetic ratio, $4\pi M$ is the saturation magnetization, and $H_{01} \approx 3753$ Oe at $4\pi M = 1800$ Oe) the power of the reflected signal falls off sharply, indicating an absorption of power upon the excitation of magnetic waves. The sample had an irregular shape with a maximum dimension ~ 3 cm, so individual resonances with a linewidth $\sim 2\Delta H$ were not observed: The various wave modes merged to form a continuous absorption spectrum with a smooth envelope. A study of the spectrum of the reflected signal showed that in fields in a narrow interval at the right-hand boundary of the absorption zone some satellite frequencies appear in the spectrum of the signal, indicating the onset of an instability. The threshold for the nonlinear effects corresponded to $P_{inc} \approx 3 \mu$W. With increasing P_{inc}, the interval of magnetic fields in which the instability was observed became broader, and the number of satellites greater. Figure 2a shows the spectrum of the

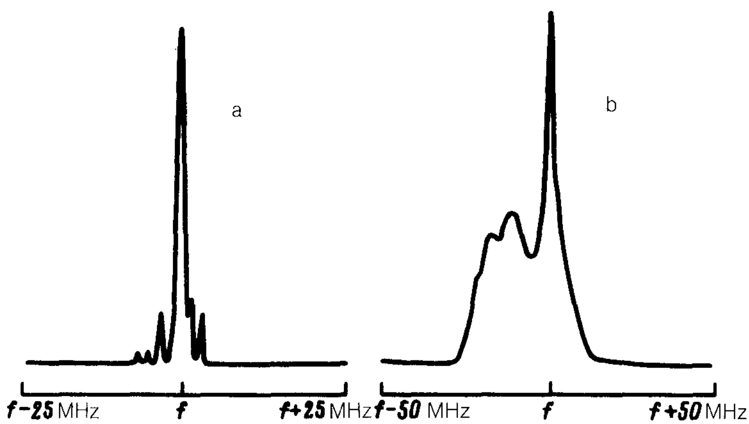

FIG. 2. Spectrum of the reflected signal for $f = 5468$ MHz. a—$P_{inc} = 10$ μW, $H_0 = 3759$ Oe; b—$P_{inc} = 1$ mW, $H_0 = 3773$ Oe.

signal at $P_{inc} = 10\mu W$. Increasing the incident power to 1 mW led to the formation of a noisy spectrum (Fig. 2b), however, and in this case the nonlinear effects were detected only at fields 3730 Oe $< H_0 <$ 3820 Oe, i.e., at the right-hand edge of the absorption band.

Discussion of experimental results. We will show that the nonlinear effects which were observed can be interpreted as a modulation instability of an electromagnetic-spin wave, i.e., of a magnetization wave of relatively large wavelength—such that retardation effects must be taken into account in analyzing such waves.

Descriptions of spin waves in magnetic films ordinarily use the magnetostatic approximation; i.e., Maxwell's equations are written in the form $\mathrm{curl}\mathbf{H} = 0$, $\mathrm{div}\mathbf{B} = 0$. The approximation is justified when all the characteristic dimensions of the test sample (its length, the length of the spin wave, etc.) are much smaller than $\lambda = c/(f\sqrt{\epsilon})$, where ϵ is the dielectric constant, and c the velocity of light. For YIG we would have $\epsilon \sim 9$ and thus $\lambda \sim 2$ cm at $f \sim 5$ GHz. The right-hand edge (along the magnetic field scale) of the absorption band corresponds to spin waves with a large wavelength λ_{sw} (in the magnetostatic approximation we would have $\lambda_{sw} \to \infty$ at $H_0 = H_{01}$). The dimensions of the sample are also greater than λ. We thus see that retardation must be taken into account in order to analyze processes in the region in which the instability is observed.

We describe the waves which arise in the sample as a superposition of traveling waves. We adopt $D(\omega, q, |\phi|^2) = 0$ as the dispersion relation for the nonlinear electromagnetic-spin waves, where q is the wave number, and $|\phi|^2$ is the square of the dimensionless wave amplitude. From the general theory of nonlinear waves in dispersive media (Ref. 1, for example) we know that the modulational instability of a wave can arise if the Lighthill condition $\omega''/(\partial\omega/\partial|\phi|^2) < 0$ is satisfied (where $\omega'' = \partial^2\omega/\partial q^2$ for $|\phi|^2 = 0$, i.e., for a linear wave). A "modulation instability" is understood here as the appearance of an envelope wave with a frequency Ω and a wave number κ. It is assumed that we have

$$\Omega \ll \omega, \quad \kappa \ll q. \tag{1}$$

In the absence of dissipation, the instability growth rate β reaches its maximum at $\kappa = \kappa_0 = \sqrt{2|\beta|/|\omega''|}$. The quantity κ_0 can be estimated under the assumption that when there is a dissipation, the instability sets in at $\beta \approx \omega_r$, where ω_r is the relaxation frequency. We then find

$$\kappa_0 = \sqrt{2|\omega_r|/|\omega''|}. \tag{2}$$

When the initial wave reaches a certain threshold amplitude, we can then expect the appearance of an envelope wave with a wave number $\kappa = \kappa_0$ if

$$\kappa_0 \ll q, \tag{3}$$

i.e., if the second condition in (1) holds. That condition allows us to use the model of a modulational instability in describing the instability. It follows from (2) that relation (3) can be satisfied in the region of the dispersion curve which has a large curvature,

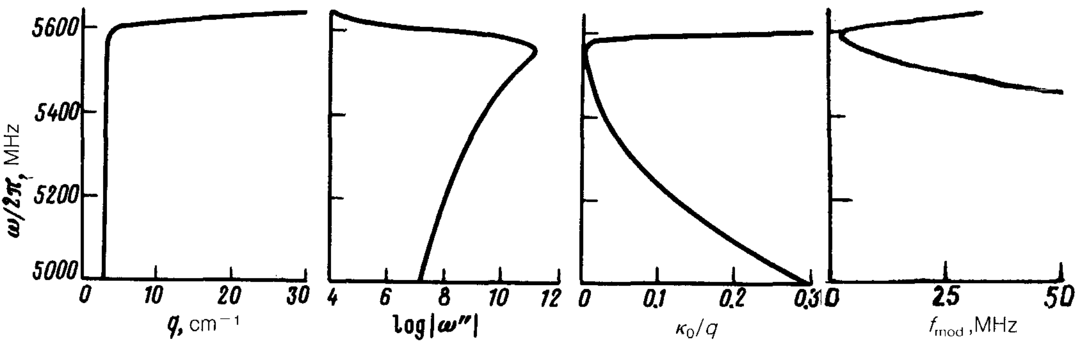

FIG. 3. Theoretical frequency dependence of q, of $\log|\omega''|$, of κ_0/q, and of $f_{\rm mod}$ for $S = 10\ \mu$m, $\omega_r/2\pi = 1$ MHz, $4\pi M = 1800$ Oe, and $H_0 = 3800$ Oe.

i.e., where $|\omega''|$ is large. Incorporating retardation makes it possible to find a region of this sort in the spectrum of natural wave modes of the ferromagnetic film.

Using the results of Ref. 2, we can construct a dispersion relation for linear electromagnetic-spin waves. In addition, we can find $|\omega''|$, κ_0, and the ratio κ_0/q by numerical methods, and we can estimate the modulation frequency, i.e., the deviation of the frequency of the satellites from the frequency of the initial wave, $f_{\rm mod} = (\kappa_0/2\pi)(\partial\omega/\partial q)$. Some results of such calculations are shown in Fig. 3. We see from this figure that there is a frequency region in the spectrum of an electromagnetic-spin wave (magnetodynamic wave) where the dispersion is very large (this region is near the lower boundary of the spectrum of magnetostatic waves). At $q \sim 3$–5 cm^{-1}, the value of $|\omega''|$ increases by several orders of magnitude, the ratio κ_0/q decreases, and the strong inequality in (3) hold well. A modulational instability with a maximum growth rate can thus develop in this region. On the other hand, it is in this region, which corresponds to the edge of the absorption band, that an instability has been observed experimentally at a threshold power $P_{\rm inc} \approx 3\ \mu$W. This figure is several orders of magnitude lower than in Refs. 3 and 4, where a study was made of nonlinear effects in the magnetostatic-wave spectrum, and the values of the ratio κ/q were 0.3 and 0.75, respectively. The calculated value of the modulation frequency of the electromagnetic-spin waves is a few megahertz, as we see from Fig. 3; this frequency agrees with the experimental results (Fig. 2a).

Note that observing a modulational instability of electromagnetic-spin waves requires the excitation of a wave which is relatively long, whose length corresponds to $q \sim 3$ cm^{-1}. The sample must therefore not be overly small, and the exciting antenna must be fairly wide.

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