NMR-echo study of topological phase in two-level system

V. N. Lisin, G. G. Fedoruk, and E. P. Khaimovich
E. K. Zavoĭskiĭ Kazan' Physicotechnical Institute, Academy of Sciences of the USSR;
A. N. Sevchenko Scientific-Research Institute of Applied Physics Problems

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A Berry-Aharonov-Anandan topological phase has been observed in a two-level system. It was created during cyclic evolution by a 2π pulse in a rotating coordinate system.

Berry[1] discovered that during the adiabatic evolution of a system along a closed curve in the parameter space of the Hamiltonian the state vector would acquire an additional geometric phase (in addition to its ordinary dynamic phase). In 1987, Aharonov and Anandan[2] extended the concept of a geometric phase to a nonadiabatic evolution. Suter *et al.*[3] observed a manifestation of a geometric phase in experiments on the nuclear spin echo in a three-level system with a nonequidistant spectrum. A cyclic evolution was arranged by means of an auxiliary 2π pulse. It was stated that a geometric phase was unobservable in two-level systems.

In the present letter we are reporting a study of a manifestation of a geometric phase in the NMR spin echo in a two-level system. It would appear at first glance that in a system of this sort a 2π pulse would not alter the observable quantities because of the 4π symmetry. The wave function would change sign under a rotation of 2π, so an observable quantity, determined by a product of wave functions, would not change. If the frequency of the 2π pulse is tuned away from the resonant frequency, however, the phase acquired in the laboratory coordinate system during the application of the 2π pulse would differ from π, with the result that there would be changes in observable quantities. To demonstrate this point, we note that in a rotating coordinate system the evolution of the state of the system upon the application of a pulsed agent has the well-known form[4]

$$|\tilde{\psi}(t)\rangle = \exp\{-i\check{H}t\}\ |\tilde{\psi}(0)\rangle = \exp\{-it\Omega\mathbf{nS}\}\ |\tilde{\psi}(0)\rangle, \tag{1}$$

where $\tilde{H} = \Omega \mathbf{n}\mathbf{S}$; $\Omega = \sqrt{(\omega_0 - \omega)^2 + \omega_1^2}$; ω_0, ω, and ω_1 are the resonant frequency, the frequency of the alternating field, and the Rabi frequency at a zero frequency deviation, respectively; $\mathbf{n}$ is a unit vector along the effective magnetic field; and $\mathbf{S}$ is the spin of a particle. At the end of the 2π pulse (i.e., at the end of a pulse of length $T = 2\pi/\Omega$), we then find $|\tilde{\psi}(T)\rangle = -|\tilde{\psi}(0)\rangle$ for $s = 1/2$ from (1). We see that the evolution of the state of the system under the influence of the 2π pulse results in the acquisition of a phase of π by the wave function in the rotating coordinate system. This phase can be broken up into dynamic and geometric parts: $|\tilde{\psi}(T)\rangle = \exp[i\gamma_d + i\gamma(C)]\,|\tilde{\psi}(0)\rangle$. If the initial state is an eigenstate for S_z ($|\tilde{\psi}(0)\rangle = |m\rangle$), then by definition we have[2]

$$\gamma_d = -\int_0^T \langle \tilde{\psi}(t)|\tilde{H}(t)|\tilde{\psi}(t)\rangle dt = -m(\omega_0 - \omega)T. \tag{2}$$

In order to calculate the geometric phase $\gamma(C)$, it is necessary to transform to a projective Hilbert space $|\tilde{\psi}'(t)\rangle$ of such a nature that we have $|\tilde{\psi}'(T)\rangle = |\tilde{\psi}'(0)\rangle$. From (1) we find that $|\tilde{\psi}'(t)\rangle = \exp(i\Omega t/2)\,|\tilde{\psi}(t)\rangle$ satisfies this condition. By definition[2] we then have

$$\gamma(C) = i \oint_{(C)} \langle \tilde{\psi}'(t)\,|d/dt|\,\tilde{\psi}'(t)\rangle dt = -mO(C), \tag{3}$$

where $O(C)$ is the solid angle which is subtended at the origin of the rotating coordinate system by the closed orbit which is traced out by the tip of the spin vector, which is initially directed along the z axis:

$$\begin{aligned} O(C) &= T(\Omega - \Omega_z) = 2\pi(1 - \cos\Theta), \\ \cos\Theta &= (\omega_0 - \omega)/\Omega. \end{aligned} \tag{4}$$

In the laboratory coordinate system we have

$$|\psi(T)\rangle = \exp\{-iT\omega S_z\}\,|\tilde{\psi}(T)\rangle = \exp\{i\gamma_m(C)\}\,|\psi_0(T)\rangle, \tag{5}$$

where $\gamma_0(T) = \exp(-i\omega_0 mT)|m\rangle$ is the wave function in the absence of a 2π pulse. We thus see that the 2π pulse gives the wave function in the laboratory coordinate system a phase which differs from π and which is equal to the geometric phase in the rotating coordinate system.

It follows from (5) that the evolution of the density operator $\rho = |m\rangle\langle m'|$ under the influence of the 2π pulse is described by

$$\rho_{mm'}(T) = \rho_0(T)\exp\{i[\gamma_m(C) - \gamma_{m'}(C)]\}, \tag{6}$$

where $\rho_0 T = \exp[-i\omega_0 T(m - m')]\,|m\rangle\langle m'|$ is the density operator at the same time in the absence of a 2π pulse. The observed echo signal is determined by the components ρ_{-+} of the density matrix ($m = -1/2$, $m' = 1/2$). It thus follows from (3), (4), and (6) that the effect of the 2π pulse reduces to changing the signal phase by an amount $\beta[\cos(\omega t + \phi) \to \cos(\omega t + \phi + \beta)]$. This phase change is determined by the topological phases of the state vectors in the rotating coordinate system:

$$\beta = O(C). \tag{7}$$

A topological phase is observable if $\omega \neq \omega_0$.

Experimentally, a geometric phase can be realized in a two-level system if one applies an additional 2π pulse in the time interval between two pulses which are separated by an interval τ and which generate an echo signal at the time 2τ. Alternatively, the additional pulse could be applied between the second exciting pulse and the echo signal. The 2π pulse could be created by a third rf pulse with a frequency equal to or different from the frequency of the pulses which generate the echo signal at the time 2τ. If the frequencies of the three rf pulses are identical, the pulses which form the 2τ echo act in a resonant fashion ($\omega = \omega_0$), and the 2π pulse is applied to the spin system when its resonant frequency has been altered by a pulse of a polarizing magnetic field. We have also realized this situation during the application of a nonresonant 2π pulse in the interval between τ and 2τ; the 2τ echo signal was observed at $\omega = \omega_0$.

The measurements were carried out at 14.4 MHz on an NMR spectrometer at room temperature for protons in water or glycerin. The pulse of polarizing magnetic field was produced by means of Helmholtz coils 15 mm in radius. The amplitude of the rf field was determined from the frequency of the transient nutations observed during the application of the pulses and from the dependence of the signal amplitude on the area under the second pulse. The deviation from the resonant frequency, $\omega_0 - \omega$, was determined from the change in the free-induction signal upon the application of the pulse of polarizing magnetic field. The same signal was used to monitor the shape of the pulse. During the formation of the echo signal, we introduced a controllable gradient in the polarizing magnetic field. The inhomogeneous linewidth was 1/

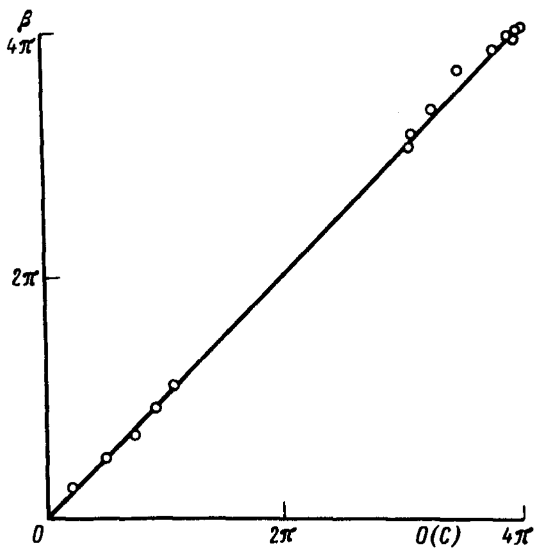

FIG. 1.

$T^* \ll (\omega_0 - \omega)2\pi$, and the pulse length was $t_i \ll T^*$. The geometric phase was determined with respect to the dynamic phase caused by the evolution of the spin system during the application of the two rf pulses and the magnetic-field pulse. This geometric phase was measured from the ratio of the amplitudes of the echo signals which were detected at the time 2τ in the orthogonal (X and Y) channels of a phase-sensitive detector. The phase of the signal in the absence of the 2π pulse was chosen equal to zero (the Y component was 0).

Figure 1 shows the measured values of β as a function of the solid angle $O(C)$. The experimental points are seen to conform quite accurately to the theoretical prediction of the behavior $\beta = O(C)$. It can be concluded that a geometric phase has been observed in a two-level system.

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