Mixing of *B* mesons and dynamic flavor

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The $SU(3)_H$ horizontal gauge interaction of chiral symmetry of the generations provides a natural explanation for the pronounced $B_d^0 - \overline{B}_d^0$ mixing in the case of a light t quark ($m_t \leqslant M_W$). The $K^0 - \overline{K}^0$ transitions are not perturbed significantly in the process. A mixing in the $B_s^0 - B_s^0$ system substantially weaker than that in the standard model is predicted.

1. In the standard model, the pronounced $B_d^0 - B_d^0$ mixing observed experimentally[1] necessarily leads to the conclusion that the mass of the t quark must be fairly large,[2] $m_t = 100$–150 GeV. On the other hand, several attempts have been made to explain this phenomenon in the case of a light t quark also ($m_t \leqslant M_w$, where M_w is the mass of a W boson), outside the standard model—on the basis of four generations of quarks, a supersymmetric expansion of the standard model, an expansion of the Higgs sector of the standard model, etc. (see the review by Ali[3]).

In the present letter we wish to discuss $B_d^0 - \overline{B}_d^0$ mixing in a model of $SU(3)_H$ chiral local symmetry of generations.[4] We will show that specifically this process determines the scale of the $SU(3)_H$ horizontal gauge interaction.

2. The model contains, in addition to the ordinary [according to $(SU(2) \otimes U(1)]$ quarks and leptons, which by assumption form $SU(3)_H$ chiral triplets,[4]

$$L_\alpha = \begin{pmatrix} u \\ d \end{pmatrix}_{L\alpha}, \quad R^\alpha = (u, d)_R^\alpha, \quad \alpha = 1, 2, 3 \ \ (SU(3)_H) \tag{1}$$

and the standard electroweak scalar φ, some horizontal Higgs multiplets

$$\xi^\alpha, \sigma^\alpha, \eta^\alpha; \ \ \chi_{\{\alpha\beta\}} \tag{2}$$

which spontaneously break $SU(3)_H$ symmetry. The Higgs potential is

$$P_H = P_0 + h\epsilon_{\alpha\beta\gamma}\xi^\alpha\sigma^\beta\eta^\gamma, \tag{3}$$

where P_0 contains standard polynomials of these fields, and the second term guarantees (for an arbitrary value of the constant h) the orthogonality of the vacuum expectation values of the ξ, σ, and η triplets. As a result, we have, in essentially the general case, expectation values for the latter and for the χ sextet of the form

$$\langle \xi^\alpha \rangle = \xi\delta_1^\alpha,\ \langle \sigma^\alpha \rangle = \sigma\delta_2^\alpha,\ \langle \eta^\alpha \rangle = \eta\delta_3^\alpha;\ \langle \chi_{\{\alpha\beta\}} \rangle = \chi\delta_\alpha^3\delta_\beta^3\ . \tag{4}$$

On the other hand, the same fields form effective Yukawa couplings of the up and down quarks:

$$\bar{L}^\alpha u_R^\beta \frac{\varphi}{M_U} \Delta^n_{\alpha\beta} G^u_n,\ \ \bar{L}^\alpha d_R^\beta \frac{\tilde{\varphi}}{M_D} \Delta^n_{\alpha\beta} G^d_n,\ \ n = 0, 1, 2 \tag{5}$$

($\Delta^0 = \chi_{\{\alpha\beta\}}, \Delta^1_{\alpha\beta} = \epsilon_{\alpha\beta\gamma}\xi^\gamma, \Delta^2_{\alpha\beta} = \epsilon_{\alpha\beta\gamma}\eta^\gamma$; the $G^{u,d}_n$ are constants), where $M_{U,D}$ are mass regulators associated with the masses of "intermediate" heavy scalars[5] or fermions.[6] Couplings (5), in contrast with the Yukawa couplings of the standard model, also have Peccei–Quinn symmetry,[7] since the corresponding charge Y can be immediately assigned horizontal scalars χ, ξ, and η ($Y_\chi = Y_\xi = Y_\eta$). On the other hand, the relation between the $U(1)_{PQ}$ charges of the triplets, $Y_\xi + Y_\sigma + Y_\eta = 0$, which follows from the Higgs potential of these fields, (3), says that one of these triplets—in our case the triplet σ^α—cannot form Yukawa couplings of the type in (5). As a result, from couplings (5) and the vacuum expectation values of the fields χ, ξ, and η in (4) we find the specific Fritzsch ansatz[8] with an antisymmetric mixing between generations for the mass matrices of the up and down quarks:

$$\hat{m}^u = S^u(\chi) + A^u(\xi, \eta),\ \ \hat{m}^d = S^d(\chi) + A^d(\xi, \eta). \tag{6}$$

The elements of these matrices, $\hat{m}_{\alpha\beta}$ ($\alpha\beta = 33,23,12$) conform to a hierarchy set by the hierarchy of expectation values, for which we adopt

$$\chi : \xi : \eta = 1 : p : p^3,\ \ \sigma : \chi \approx 1 : p\ \ (p \approx 0.15), \tag{7}$$

working in the first case from the observed quark mass spectrum and in the second from the condition $\Delta M_K^H \lesssim \Delta M_K^W$ (§3).

3. We now consider the dynamic consequences of the model. By virtue of the chiral content of fermions [the left-hand quarks are triplets, and the right-hand quarks antitriplets; see (1)], the gauge couplings of the $SU(3)_H$ horizontal bosons are

$$\frac{1}{2} g_H H^A_\mu [\bar{L}\lambda_A \gamma_\mu L - \bar{R}\lambda_A^T \gamma_\mu R]\ \ A = 1, \ldots, 8 \tag{8}$$

where λ_A are the Gell–Mann matrices. The H^A boson mass spectrum is determined by essentially only two scalars of set (2), by the triplet σ^α, and by the sextet $\chi_{\{\alpha\beta\}}$, which develop the largest vacuum expectation values in the model [see (7);$\chi^2 = p^2\sigma^2 \ll \sigma^2$]:

$$M^2_{1,2,6,7} = \frac{g_H^2}{2}\sigma^2,\ \ M^2_{4,5} = g_H^2\chi^2,\ \ M^2_{8-3} = g_H^2\sigma^2,\ \ M^2_{8+3} = 4g_H^2\chi^2, \tag{9}$$

where $M^2_{8\pm3}$ corresponds to two superpositions of bosons, $H^{8\pm3} = H^8/\sqrt{3} \pm H^3/\sqrt{2}$.

The chiral gauge couplings of massive H bosons lead to effective four-fermion interactions which do not conserve the quark flavor either through exchanges of "charged" bosons $(H^A \pm iH^{A+1}/\sqrt{2})$ $(A = 1, 4, 6)$ or, after a diagonalization of the massive quarks $\hat{m}^u$ and $\hat{m}^d$ in (6), through exchanges of "neutral" bosons $H^{8\pm3}$. In the sector of down quarks we have the following results for the matrix elements of the transitions $K^0 - \overline{K}^0$, $B_d^0 - \overline{B}_d^0$, and $B_s^0 - \overline{B}_s^0$:

$$T_K \approx [\frac{2}{\sigma^2} e^{i\alpha} + \frac{1}{\chi^2} (\frac{1}{4} s_1^2 + s_2^2 e^{i\beta})]\rho_K O_{LL}^K,$$

$$T_{B_d} \approx [\frac{1}{\chi^2} e^{i\beta}]\rho_B O_{LL}^{B_d}, \quad T_{B_s} \approx [\frac{2}{\sigma^2} e^{i\gamma} + \frac{1}{\chi^2} s_1^2 e^{i\beta}]\rho_B O_{LL}^{B_s} \cdot \qquad (10)$$

From $T_P = \langle P | - \mathscr{L}_{eff} | \overline{P} \rangle (P = K, B_d, B_s)$ we are omitting small terms on the order of $(\rho_K/\sigma^2)s_{1,2}^2$, $(1/4\chi^2)s_{1,2}^2$, $(\rho_K/\chi^2)s_1^2 s_2^2$, etc. For the brackets of the products of left-hand currents, O_{LL}^P, right-hand currents, O_{RR}^P, and left- and right-hand currents, O_{LR}^P, we have adopted everywhere in (10) the following in the approximation of a vacuum "interleaving":

$$O_{RR}^P = O_{LL}^P; \quad O_{LR}^P = -\rho_P O_{LL}^P, \quad \rho_{B_d} \approx \rho_{B_s} \approx 1.4 \quad (m_b = 4.7\,\mathrm{GeV}) \qquad (11)$$

and $\rho_k = 5.8$ for $m_s = 150$ MeV (Ref. 9). The sines s_1 and s_2 $(c_1 \approx c_2 \approx 1)$ and the phases α, β, and γ are related to the diagonalization of the mass matrix $\hat{m}^d$. The latter arise only in products of "charged" currents with generators $\lambda_1 \pm i\lambda_2$ (the α phase), $\lambda_4 \pm i\lambda_5$ (the β phase), and $\lambda_6 \pm i\lambda_7$ (the γ phase). The diagonal generators λ_3 and λ_8 commute with the phase matrices of the quarks. For a matrix $\hat{m}^d$ of general form, (6), with arbitrary complex elements, we have

$$s_1 = (m_d/m_s)^{1/2}, \quad s_2 = (m_s/m_b)^{1/2}; \quad \alpha = \pi, \quad \beta = 0, \quad \gamma = \pi. \qquad (12)$$

From expressions (10) with phases (12) follow the two basic results of the present study. First, the $SU(3)_H$ horizontal gauge interaction automatically conserves CP invariance in the transitions $K^0 - \overline{K}^0$, $B_d^0 - \overline{B}_d^0$, and $B_s^0 - \overline{B}_s^0$. Second, the contributions to the mass difference ΔM_K from the exchange of "isotopic" bosons, $(H^1 \pm iH^2)/\sqrt{2}$, and U-spin bosons, $(H^4 \pm iH^5)/\sqrt{2}$, have opposite signs and cancel each other out, while the total contribution to the mass difference ΔM_{B_d} is due to U-spin bosons, and there is no such cancellation.

Let us look at specifically $B_d^0 - \overline{B}_d^0$ mixing. For the Fritzsch ansatz, (6),

$$|V_{td}| \approx (m_d/m_s)^{1/2} |V_{ts}| \approx (m_d/m_s)^{1/2} |V_{cb}| \approx 0.01 \quad (|V_{cb}| = 0.045), \qquad (13)$$

the usual weak "box" with a t-quark mass $m_t = 70$ GeV and a constant $B^{1/2} f_B = 0.12$ leads to a mixing parameter $x_d^W = 0.12$, which is much smaller than the observed value,[1] $x_d = 0.73 \pm 0.18$. Taking the contribution of the horizontal interaction to be $x_d^H = 0.6$, we find from T_{B_2} in (10) its scale:

$$x_d = x_d^W + x_d^H = 0.72, \quad \chi = 450 \text{ TeV}. \tag{14}$$

From this scale we can find the scale of σ, requiring a complete or partial cancellation in T_K in (10) of the contributions from different bosons. If the cancellation is complete, this scale turns out to be [here we are using angles (12) with $m_d/m_s = 1/20$, $m_s/m_b = 1/35$, and the parameter value $\rho_K = 5.8$] $\sigma \approx 3000$ GeV. This figure corresponds to the hierarchy of vacuum expectation values which we have adopted, (7). Because of the large scale of χ (and σ) in the model, all other processes in which the quark and/or lepton flavor is not conserved must also proceed extremely slowly. The decay "allowed" to the greatest extent in this model, $b \to d + \tau + \tilde{e}$, for example, goes with a relative weight $\sim 10^{-10}$.

With regard to $B_s^0 - \overline{B}_s^0$ mixing, one can see from T_{B_s} in (10) and the values of the χ and σ scales that the contribution of the horizontal interaction to this mixing is negligible and that this mixing is governed essentially completely by the ordinary weak "box." An important signature for it is the circumstance that it is now substantially smaller than in the standard model with large m_t:

$$x_s \approx x_s^W = \frac{x_d^W}{x_d} x_s^{st} = R x_s^{st}, \tag{15}$$

where the ratio R in the general case—for a t-quark mass $m_t = 70$ GeV, $x_d = 0.7$, and $|V_{td}| \leqslant 0.02$—has an upper limit $R \leqslant 0.6$. For the Fritzsch ansatz which we have been discussing [$|V_{td}| = 0.01$, $x_d^W = 0.12$; see (13)], this ratio is $R \approx 1/6$. Numerically, for $m_t = 70$ GeV we find the value $x_s \approx 2.4$. The experimental observation of a value of x_s close to this value, along with information that there is no fourth generation of quarks (the only model, of which I am aware, in which such small values of x_s are possible in principle[3]), might provide a serious argument in favor of the approach proposed here.

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