Temperature–(magnetic field) phase diagram for type-II superconductor near critical point

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Arguments are presented to support the suggestion that the phase transition to the normal state in a type-II superconductor is due to a spontaneous nucleation of Abrikosov vortices and is a first-order transition. The effect of defects on this transition is determined.

The nature of the phase transition to the normal state in type-II superconductors is not yet completely clear. Halperin *et al.*[1] have concluded that this transition is abrupt (as in type-I superconductors), but the numerical analysis by Dasgupta and Halperin[2] led to the conclusion that the transition is continuous. The analysis carried out in those papers is not complete: Halperin[1] ignored phase fluctuations, and Dasgupta and Halperin[2] ignored fluctuations in the amplitude of the order parameter. It was found in Refs. 3 and 4 that there is a point on the phase diagram at which the magnetic-field penetration depth δ is comparable to the correlation length ξ. They interpreted that point as a transition to type-I superconductivity (it is true that the corresponding values of ζ in those studies were slightly different).

The temperature–(magnetic field) phase diagram ($T-H$ diagram) which follows from the results of Refs. 3 and 4 appears to be impossible on the basis of simply continuity: Two lines of second-order phase transitions merge into a line of first-order transitions with a finite discontinuity at the point of merging.

This contradiction can be eliminated by assuming that at $H=0$ the phase transition to the normal state involves the spontaneous nucleation of Abrikosov vortices at a finite ζ. Since vortices of different signs attract each other, a transition of this sort will unavoidably be of first order. The corresponding phase diagram has the form shown in Fig. 1. Near T_1 the thermodynamic critical field H_c vanishes in a root fashion, since the transition is abrupt. From continuity conditions at the triple point we have $dH_c/dT=0$. The vertical line which connects the $H_{c_1}(T)$ and H_{c_2} curves has been drawn somewhat arbitrarily; its position is actually not known.

We wish to present some arguments in favor of this suggestion. The idea that a phase transition from a superfluid state to the normal state in liquid helium involves the spontaneous nucleation of vortices was advanced more than 30 years ago by Feynman.[5] Actually, however, the free energy of a vortex vanishes only at $\xi=\infty$. This circumstance seems completely natural in light of the principle of the universality of critical phenomena: In this case it is not possible to cite any characteristic line, other than $\xi=\infty$, which might associate ξ at the point of the phase transition with a spontaneous nucleation of vortices. On the other hand, there is such a length, $\xi^*\sim\Phi_\sigma^2/T_c$ in a superconductor[4] (Φ_0 is the magnetic flux quantum, and T_c is the

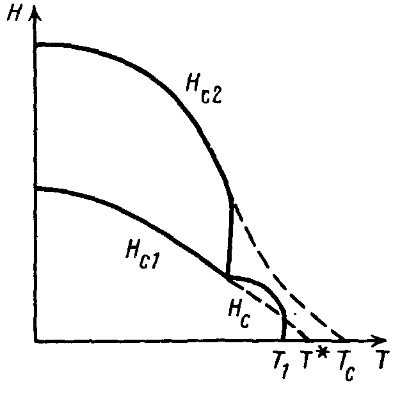

FIG. 1.

phase-transition temperature when the charged nature of the superconducting fluid is ignored).

An indication that the free energy of a vortex can vanish at $\xi \sim \xi^*$ can be found even in linear elastic theory for an isolated vortex. For the free energy per unit length of an isolated vortex, f, we have, with logarithmic accuracy,

$$f = \epsilon - C_1 \epsilon \langle \left(\frac{du}{dz} \right)^2 \rangle = \epsilon - C_1 \epsilon \int \frac{dk}{2\pi} k^2 \langle \mathbf{u}_k^2 \rangle = \epsilon - C_2 T/\xi. \tag{1}$$

Here $\mathbf{u}$ is the displacement vector of a point of the vortex for a displacement parallel to the z axis, and C_1 and C_2 are constants on the order of unity. The integration over k is limited by $k_{\max} \sim 1/\xi$, i.e., by the reciprocal thickness of the vortex core. The latter relation can be derived through an analysis of the spectrum of oscillations of the vortex as in Ref. 6. Using the definition of the linear tension of the vortex, $\epsilon = (\Phi_0/4\pi\delta)^2 \ln \kappa$, where $k = \delta/\xi$, we find

$$f = \frac{T}{\xi} \left(\frac{\Phi_0^2 \xi}{16\pi^2 \delta^2 T} \ln \kappa - \text{const} \right). \tag{2}$$

According to the Josephson relation,[7] the coefficient of the logarithm is a constant on the order of unity near the transition point, and $\ln \kappa$ decreases, as was shown in Refs. 3 and 4. At $\xi \sim \xi^*$, the expression in parentheses in (2) may vanish. At $\xi \sim \xi^*$, however, the contributions to the free energy from the nonlinear terms in the Hamiltonian of an isolated vortex are also on the order of T/ξ. As a result, the sign of the coefficient of T/ξ cannot be determined. Furthermore, the very expression for the vortex energy written under the assumption $\ln \kappa \gg 1$ becomes inapplicable. Nevertheless, the topological considerations which we cited at the beginning of this paper require a vanishing of the free energy of the vortex even at a finite ξ.

As we already mentioned, a phase transition accompanied by the nucleation of vortices must be a first-order transition. It would naturally occur at a temperature lower than that at which the free energy of an isolated vortex would vanish. The jump in the density of the superconducting component, n, can be estimated by substituting

$\xi = \xi^*$ into the Josephson relation. We find

$$n \sim mT_c^2 e^2/\hbar^4 C^2 \sim \frac{m}{m_0} \left(\frac{T_c}{T_{at}}\right)^2 \alpha^2 N_{at}, \tag{3}$$

where m and m_0 are the effective and actual masses of an electron, N_{aT} is the atomic electron density, T_{aT} is a characteristic atomic temperature ($\sim 10^4$ K), and $\alpha = 1/137$ is the electrodynamic constant. It can be seen from (3) that even at $T_c \sim 100$ K the value of n is $10^{-8} N_{aT}$; i.e., it will be difficult to observe experimentally.

The presence of defects constituting inclusions of the normal phase in a crystal would lead to an additional lowering of the energy of a vortex. It follows from Ref. 8 that the defect concentration c appears in the expression for the free energy in the combination $c\xi^3$. This circumstance means that an additional length $\tilde{\xi} \sim c^{-1/3}$ appears in the system, and the energy of the vortex can change sign at $\xi \sim \tilde{\xi}$. In other words, a transition with a spontaneous nucleation of vortices can be caused by defects. This will be the case if $\tilde{\xi} \ll \xi^*$. The nature of the anomalies is different from that in the pure material. Under the opposite inequality, defects will cause simply a slight lowering of the transition temperature. With an increase in the defect concentration, $\tilde{\xi}$ may become smaller than the characteristic size of a defect, and the nature of the transition may change. It is quite possible that this will be a second-order transition: Boyanovsky and Cardy have shown[9] by the $4\text{-}\epsilon$ expansion method that a phase transition to the normal state in a type-II superconductor with a high impurity concentration is a second-order transition.

Even at a concentration $c = 10^{12}$ cm^{-3} and for a transition temperature $T_c \sim 100$ K in the defect-free crystal we would have $\tilde{\xi}/\xi^* \sim 10^{-4}$; i.e., the anomalies near the transition would be determined by the presence of defects.

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