Domains of phase deconfinement in lattice gluodynamics

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The domains of phase deconfinement in $SU(2)$ lattice gauge theory have a noninteger dimensionality. Near the confinement-deconfinement phase transition, the logarithm of the volume of the domains is equal to the logarithm of the area of the domains raised to a power of 1.06.

A finite temperature can be introduced in lattice gauge theory if one considers an asymmetric lattice of size L^3N_t. In this case the temperature is related to the number of steps of the lattice along the time direction, N_t, by $T = 1/(N_t a)$. Here a is the length of an edge of the lattice, which depends on the seed charge g. By varying g or N_t, one can thus examine the theory at various temperatures.

We have analyzed an $SU(2)$ lattice gauge theory without quarks on a lattice on size $8^3\times 4$. In this theory, a confinement-deconfinement phase transition[1] occurs at

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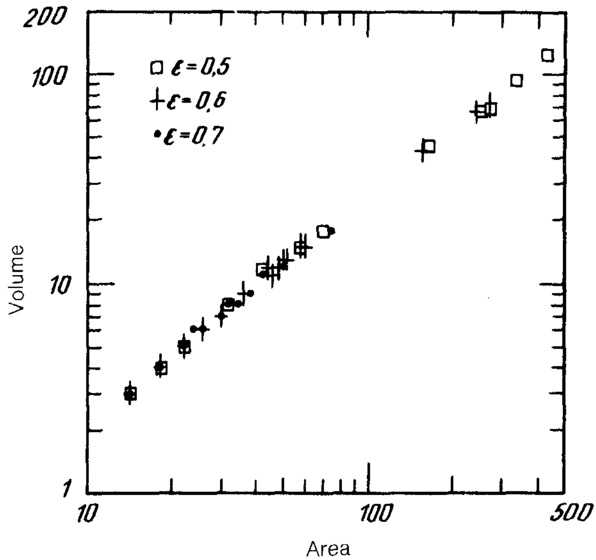

FIG. 1. Area dependence of the volume of deconfinement domains.

$4/g_c^2 \approx 2.32$. The order parameter in this case is the Polyakov-Wilson line L, the expectation value of which is related to the free energy of the color charge (of a quark): $\langle L \rangle = \exp(-F)$. In the confinement phase we thus have $\langle L \rangle = 0$, and at the phase transition we have $\langle L \rangle \neq 0$. Consequently, a value of L is associated with each vertex of the spatial lattice. A nonzero expectation value $\langle L \rangle$ arises because the distribution of the values of L is asymmetric. If L is sufficiently large ($L > \varepsilon$, where ε is a parameter) at a given spatial point, then this point definitely belongs to the deconfinement phase. We are assuming by definition that the deconfinement phase in this case occupies at least the corresponding elementary cube of the associated dual lattice; the point under consideration lies at the center of this cube. If two cubes have a common face, we assume that they belong to the same domain. With each configuration of fields we can thus associate a certain number of 3D domains of the deconfinement phase. In general, their number and structure are determined by the value of the parameter ε, but it turns out that a variation of this parameter over a wide range has no substantial effect on the results.

We have examined several lattice configurations of fields in the vicinity of the phase transition, and we have measured the volumes and areas of the deconfinement domains. The results for the case $4/g^2 = 2.29$ are shown in Fig. 1. The area dependence of the volume is described well by $V = \text{const} \cdot S^\alpha$, where $\alpha \approx 1.06$. For $4/g^2 = 2.29$ ($T < T_c$) and $4/g^2 = 2.35$ ($T > T_c$) the results are the same, within the errors. If the domains were instead normal 3D entities, then we would obviously have $\alpha = 1.5$ in this case. Our results shows that the domains have a more "porous" structure. Corresponding calculations[2] carried out along with a phase transition in the 3D Ising model also lead to a nontrivial exponent: $\alpha_1 = 1.15 \pm 0.05$. These numbers should be compared with the value $\alpha = 1.03 \pm 0.03$ found for $g^2 = \infty$, i.e., for values of L distributed at random between -1 and 1. We wish to stress that at $g^2=\infty$ the regions with $L>\varepsilon$ do not have the meaning of deconfinement domains.

It is interesting to supplement these calculations with an examination of the fractal characteristics of the domains. Preliminary data from measurements of the fractal dimensionality D_f, by the method of Ref. 3, reveal $1<D_f<2$.

At present, A. I. Veselov and I are carrying out a detailed study of the quantities discussed above over a wider temperature range on larger lattices. Preliminary data show that α depends only weakly on the temperature and remains essentially constant as we go from a lattice size of $8^3\times 4$ to $16^3\times 4$.

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