s-d-exchange resistive mechanism for spin-wave amplification in ferromagnetic semiconductors in rf electric field

P. E. Zil'berman, N. I. Polzikova, and A. O. Raevskiĭ
Institute of Radioengineering and Electronics, Academy of Sciences of the USSR

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If a ferromagnetic semiconductor is in a sufficiently strong, uniform rf electric field, which acts on the electrons in a resistive regime, a spin wave propagating through the semiconductor may be amplified in a nonresonant fashion (over a wide frequency range) by field energy transferred to the wave by the mechanism of *s-d* exchange.

We have studied an unbounded homogeneous and isotropic ferromagnetic semiconductor of the $HgCr_2Se_4$ type at a temperature $T < T_c$ in a static saturating magnetic field $\mathbf{H}_0$ and in an rf electric field $\mathbf{E} = \mathbf{E}_0 \cos \Omega t$, directed at an arbitrary angle θ from $\mathbf{H}_0$. In this semiconductor we studied a spin wave with a frequency $\omega(\mathbf{q})$ and a wave vector $\mathbf{q}$ traveling in the direction of the field $\mathbf{H}_0$ $(\mathbf{q}\|\mathbf{H}_0)$, along the z axis. The spin wave interacts with electrons through the mechanism of *s-d* exchange.[1] To describe this interaction, we start from the system of coupled equations consisting of the precession equation for the circular magnetization $M^{\pm}(\mathbf{r},t)$ and the kinetic equation for the electron distribution function $f_{\sigma\sigma'}(\mathbf{p},\mathbf{r},t)$, where $\mathbf{r}$ and t are the radius vector and the time, $\mathbf{p}$ is the momentum of an electron, and σ and σ' are spin indices (which take on the values $\uparrow$ and $\downarrow$). These equations were derived in Ref. 2; they take the following form for the problem at hand, in the linear approximation in the wave amplitude:

$$\frac{\partial M^+}{\partial t} = -i\gamma\{M^+[H_0 + AV^{-1}\sum_{\mathbf{p}}(f_{\uparrow\uparrow}(\mathbf{p}) - f_{\downarrow\downarrow}(\mathbf{p}))] - M_0[\alpha\Delta M^+ + 2AV^{-1}\sum_{\mathbf{p}} f_{\uparrow\downarrow}(\mathbf{p})]\} \qquad (1)$$

$$[\hbar(\frac{\partial}{\partial t} + \mathbf{v}\vec{\nabla} + e\mathbf{E}_0\cos\Omega t\frac{\partial}{\partial \mathbf{p}}) + i\phi_0]f_{\uparrow\downarrow}(\mathbf{p}) = iAM^+[f_{\uparrow\uparrow}(\mathbf{p} - \frac{\hbar\mathbf{q}}{2}) - f_{\downarrow\downarrow}(\mathbf{p} + \frac{\hbar\mathbf{q}}{2})] \qquad (2)$$

$$(\frac{\partial}{\partial t} + e\mathbf{E}_0\cos\Omega t\frac{\partial}{\partial \mathbf{p}})f_{\sigma\sigma}(\mathbf{p}) = I[f^{i}_{\sigma\sigma}(\mathbf{p})] - \frac{f^{a}_{\sigma\sigma}(\mathbf{p})}{\tau}, \qquad (3)$$

where $\gamma = g\mu_B/\hbar$ is the gyromagnetic ratio, we have a factor $g \approx 2$, μ_B is the Bohr magneton, M_0 is the saturation magnetization, A and α are the s-d exchange constant and the intralattice inhomogeneous-exchange constant, $v = \mathbf{p}/m$, m and e are the mass and charge of an electron, $\phi_0 = 2AM_0$ is the exchange splitting in the energy of the spin subbands, V is the normalization volume, I is the isotropic part of the collision integral, τ is the relaxation time of the electron momentum, $f_{\sigma\sigma} = f^i_{\sigma\sigma} + f^a_{\sigma\sigma} f^i_{\sigma\sigma}$, and $f^a_{\sigma\sigma}$ are the isotropic and anisotropic parts of $f_{\sigma\sigma}$. We are assuming $\phi_0\tau \gg \hbar$ and that the field $\mathbf{E}_0$ is uniform. In actuality, the field varies over the skin thickness l_{sk} or over the wavelength λ. If the electron displacement (l) in the field $\mathbf{E}$, which is $eE_0/m\Omega^2$ at $\Omega\tau \gg 1$ or $\sim eE_0\tau/m\Omega$ at $\Omega\tau \ll 1$, is less than l_{sk} and λ, then our assumption of a uniform field is justified. In a uniform field $\mathbf{E}_0$ the function $f_{\sigma\sigma}(\mathbf{p})$ found from (3) is also uniform.

We seek a solution of Eqs. (1)–(3) in the form

$$\begin{pmatrix} f_{\uparrow\downarrow}(\mathbf{p},\mathbf{r},t) \\ M^+(\mathbf{r},t) \end{pmatrix} = e^{i(\mathbf{qr} - \omega t)} \sum_{n=-\infty}^{+\infty} e^{in\Omega t} \begin{pmatrix} f^n_{\uparrow\downarrow}(\mathbf{p}) \\ M^+_n \end{pmatrix}$$

$$f_{\sigma\sigma}(\mathbf{p},t) = \sum_{n=-\infty}^{+\infty} e^{in\Omega t} f^n_{\sigma\sigma}(\mathbf{p}). \tag{4}$$

Substituting (4) into (1)–(3), and equating the coefficients of like harmonics, we find a system of linear and homogeneous equations for the amplitudes. From the vanishing of the determinant of this system, found in an order no higher than E_0^2, we find the dispersion relation for the spin waves which we need. This equation makes it possible, in particular, to find all of the previously known results on the electron absorption of spin waves in the case $\mathbf{E}_0 = 0$ (Ref. 2) and in the case $\mathbf{E}_0 \neq 0$, but in the collisionless regime, with $\Omega\tau \gg 1$ (Ref. 3). In addition, it allows us to find the electron absorption of the spin waves in the case $\mathbf{E}_0 \neq 0$ in the resistive regime, with $\Omega\tau \ll 1$, which has not been studied previously. If we wish to study the effect of the field $\mathbf{E}_0$ as a correction in this regime, and if we wish to find this correction from the dispersion relation by perturbation theory, we must impose the auxiliary conditions

$$\left(\frac{eE_0\tau}{mv}\right)^2 \ll 1, \quad \frac{|e\mathbf{E}_0\mathbf{q}|}{2m\phi_0^2}\phi_0\tau \ll \hbar, \quad \frac{|e\,\mathbf{E}_0\mathbf{q}|}{2m\Omega^2}\Omega\tau\delta \ll 1, \quad \delta \equiv \frac{g\mu_B(N_\uparrow - N_\downarrow)}{2M_0} \ll 1, \tag{5}$$

where $N_\uparrow$ and $N_\downarrow$ are the electron densities in the $\uparrow$ and $\downarrow$ subbands. We then find the following dispersion relation for the spin waves:

$$\omega(\mathbf{q}) = \tilde{\omega}_H(\mathbf{q}) - \frac{\phi_0}{\hbar}\frac{\mu_0^+}{M_0^+}, \tag{6}$$

where

$$\tilde{\omega}_H(\mathbf{q}) = \omega_H + \omega_m\frac{\alpha q^2}{4\pi} + \frac{\phi_0}{\hbar}\frac{g\mu_B(N_\uparrow - N_\downarrow)}{M_0}, \tag{7}$$

$\omega_H = \gamma H_0$, $\omega_m = 4\pi\gamma M_0$, and $\mu_0^{\pm}$ are the zeroth ($n = 0$) harmonics of the circular components of the magnetization of the conduction electrons. A calculation of the imaginary part of the ratio μ_0^+/M_0^+ leads to the final result

$$\mathrm{Im}\,\omega(\mathbf{q}) = 3\tau\left(\frac{\hbar e \mathbf{E}_0 \mathbf{q}}{2m\phi_0}\right)^2 \frac{g\mu_B(N_\uparrow - N_\downarrow)}{M_0}. \quad (8)$$

Substituting into (8) the typical parameter values $\phi_0 \sim 0.1$ eV, $\Omega \sim 3\times10^{11}$ s^{-1}, $\tau \sim 10^{-12}$ s, $N_\uparrow - n_\downarrow \sim 10^{20}$ cm^{-3}, $M_0 \sim 200$ G, and $E_0 \sim 8$ kV/cm with $q \sim 10^6$ cm^{-1} and $\theta = 0$, we find the estimate Im $\omega(\mathbf{q}) \sim 4.8\times10^7$ s^{-1}. This result is higher than the dissipative magnetic loss, which is $2\gamma\Delta H \sim 3.6\times10^7$ s^{-1} at a resonance-line width $2\Delta H \sim 2$ Oe. An instability thus arises. According to the Bers–Briggs criteria,[4] this instability is convective; i.e., it leads to an amplification of spin waves. Assuming $\alpha = 1.2\times10^{-12}$ cm^2, we can estimate the group velocity of the spin waves $\partial\tilde{\omega}_H/\partial q$, on the basis of (7). The spatial growth rate Im q can then be estimated to be $(\mathrm{Im}\,\omega - 2\gamma\Delta H)(\partial\tilde{\omega}_H/\partial q)^{-1} \sim 40$ cm^{-1}, and over a spin-wave propagation distance ~ 500 μm we find the amplification to be ~ 17 dB. These estimates are illustrated in Fig. 1, from which we see that at a fixed frequency $\omega > \omega_0$ the field must be above a threshold. This threshold is lowest at the angle $\theta = 0$, and it initially decreases with increasing ω. The upper boundary of the amplification region along the frequency scale apparently arises because of an increase in the linewidth $\Delta H(\omega)$ with increasing ω (Ref. 5). The band of amplified frequencies can be wide: $\Delta\omega \gtrsim \omega_0$.

The amplification mechanism is closely related to the expression for the power (W) transferred from the electrons to the lattice by s-d exchange. Here we have[1]

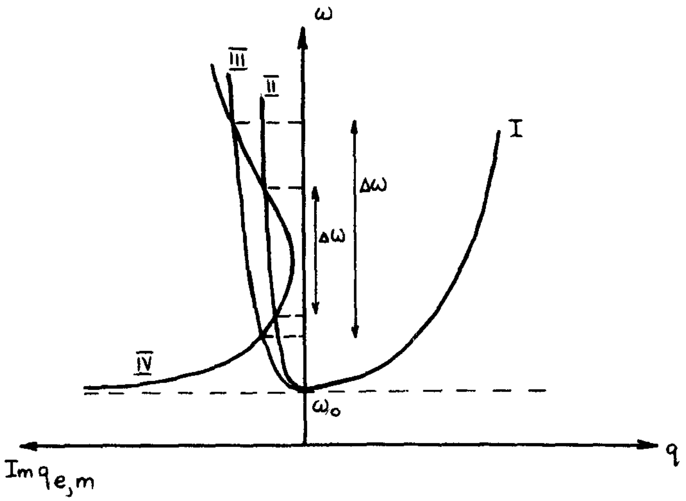

FIG. 1. I—Plot of $\tilde{\omega}_H$ versus $\mathbf{q}$ according to expression (7); II, III—plot of Im $q_e \equiv \mathrm{Im}\,\omega/(\partial\tilde{\omega}_H/\partial q)$ versus ω for two values of the field ($E_0' > E_0$); IV—plot of ($-$ Im $q_m = 2\Delta H(\omega)/\partial\tilde{\omega}_H/\partial q$) versus ω (Im q = Im q_e + Im q_m).

$W \propto (M^+\mu^- + M^-\mu^+)$, and since we have $\mu^+ \propto f_{\uparrow\downarrow} \propto M^+$ [see (2), for example] and $\mu^- \propto f_{\downarrow\uparrow} \propto M^-$, then we have $W \propto M^+M^- = |M|^2$. In other words, W is proportional to the energy density of the spin waves. In the resistive regime ($\Omega\tau \ll 1$) the electron current density **j** and the field **E** are in phase, so that, on the average, the electrons acquire from the field a Joule power $P = (1/2)\mathrm{Re}(\mathbf{j}\cdot\mathbf{E}^*) > 0$. A fraction W of this power is transferred to the spin waves more efficiently, the greater the spin-wave energy which has already built up. The stage is thus set for the onset of an avalanche process: an instability.

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[1]É. L. Nagaev, *Physics of Magnetic Semiconductors*, Nauka, Moscow, 1979.
[2]Yu. V. Gulyaev *et al.*, Fiz. Tverd. Tela (Leningrad) **26**, 2689 (1984) [Sov. Phys. Solid State **26**, 1629 (1984)].
[3]Yu. I. Balkareĭ and É. M. Épshteĭn, Fiz. Tverd. Tela (Leningrad) **14**, 81 (1972) [Sov. Phys. Solid State **14**, 65 (1972)].
[4]E. M. Lifshits and L. P. Pitaevskiĭ, Physical Kinetics, Pergamon, Oxford, 1981.
[5]A. G. Gurevich, *Magnetic Resonances in Ferrites and Antiferromagnets*, Nauka, Moscow, 1973.

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