Four-loop β function of supersymmetric Wess-Zumino–Witten model

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A four-loop renormalization-group β function is calculated for an $N = 1$ supersymmetric 2D Wess-Zumino–Witten model.

The Wess-Zumino–Witten model[1,2] (WZWM), which is widely used in applications, is a 2D nonlinear σ model on a group manifold G with a Wess-Zumino–Witten term (or twisting).[3] Its supersymmetric generalization[4] (SWZWM) is an $N = 1$ supersymmetric 2D nonlinear σ model with twisting and with an action[3]

$$I_{SWZWM} = \frac{1}{4i\lambda^2} \int d^2x d^2\theta \, [g_{ab}(\varphi) - \frac{2}{3} h_{ab}(\varphi)] \bar{D}\varphi^a (1 + \gamma_3) D\varphi^b , \quad (1)$$

in an $N = 1$, $d = 2$ plane superspace with the coordinates (x^μ, θ^α), where g_{ab} is the metric on G, h_{ab} is the twisting potential $(H_{abc} \equiv \partial_{[a} h_{bc]})$, and $(\bar{D}, D)$ are supercovariant derivatives.[1)]

The SWZWM is characterized completely by the curvature tensor R_{ijkl}, the generalized curvature tensor $\hat{R}_{ijkl}$ with twisting, and the twisting tensor itself, H_{ijk}, in the space tangent to G (Ref. 3):

$$\begin{aligned} R_{ijkl} &= f_{ij}{}^m f_{mkl} , \\ \hat{R}_{ijkl} &= (1 - \eta^2) R_{ijkl} , \\ H_{ijk} &= \eta f_{ijk} , \end{aligned} \quad (2)$$

where f_{ijk} are structure constants of the compact and semisimple Lie group G, and

$$\eta \equiv n\lambda^2 / 2\pi . \quad (3)$$

Here λ^2 is a coupling constant of the theory, and n is the coefficient of the Wess-Zumino–Witten term, which takes on only integer values by virtue of the known topological quantization effect.[2,3] The SWZWM Lagrangian is given explicitly in Ref. 4; relations (2) are sufficient for actual calculations of the β function.

For quantum-mechanical calculations by perturbation theory in nonlinear σ models, it is convenient to use a covariant background-field method, after defining a background-quantum expansion of action (1) along geodesics.[3] In this manner we find covariant Feynman rules for supergraphs.[5] By virtue of the renormalizability of the theory, the covariant l-loop counterterm is

$$I_c^{(l)} = \frac{1}{4i\lambda^2} \int d^2x d^2\theta \sum_{k=1}^{l} \frac{1}{(2\epsilon)^k} T_{a\,b}^{(k,\,l)} \bar{D}\varphi^a (1+\gamma_3) D\varphi^b , \tag{4}$$

The β functions of the σ model are determined by the residue at the simple pole,

$$\beta_{ab} \equiv \beta^g_{(ab)} + \beta^h_{[ab]} = \sum_{l=1}^{\infty} l T_{ab}^{(1,\,l)} \quad . \tag{5}$$

An UV regularization of the theory is carried out through an analytic continuation of the momentum integrals to a dimensionality $d = 2 - 2\epsilon$. The D algebra and the λ matrices are defined in $d = 2$. This is the content of the known supersymmetric dimensional regularization through dimensional reduction.[6] The IR and UV infinities are separated by means of the mass parameter m^2. Ketov[7] has calculated the three-loop β function of the WZWM in the boson case. Braaten *et al.*[3] have calculated the single-loop β function of the SWZWM in the supersymmetric case. According to the general results,[5] there are no two-loop or three-loop corrections in the supersymmetric case.

The four-loop correction to the SWZWM β function is determined by the ten basic diagrams in Fig. 1. Actually, there are 35 four-loop diagrams which make nonvanishing contributions to the β function when the specification of the vertices in Fig. 1 is taken into account.

Significantly, the infinities of all the diagrams in this figure can be reduced by means of the D algebra and an integration by parts to the infinities of the first diagram (*). This result is a generalization of the known result[8] to incorporate twisting. The corresponding integral takes the following form after a subtraction of all of the subinfinities[8]:

$$A_4 = \int \frac{d^dk d^dq d^dr d^dt}{(2\pi)^{4d}} \frac{k\cdot(t-k) q\cdot(t-q)}{k^2(t-k)^2 q^2(t-q)^2(r^2+m^2)[(t-r)^2+m^2]}$$

$$\rightarrow \frac{4}{(4\pi)^4} \left[\frac{\zeta(3)}{\epsilon} \quad \frac{1}{6\epsilon^4} \right] . \tag{6}$$

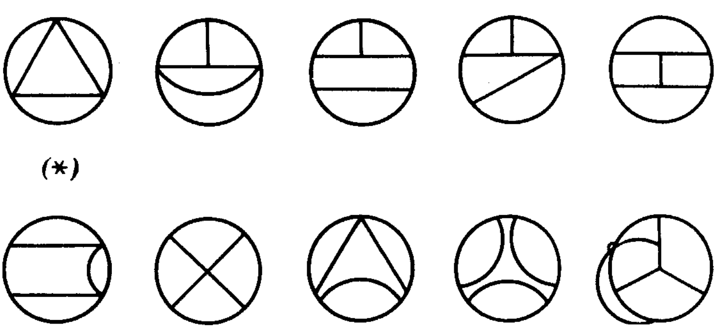

FIG. 1. Topologies of the diagrams which determine the four-loop β function of the supersymmetric Wess-Zumino–Witten model.

All contributions to the four-loop β function of an *arbitrary* supersymmetric 2D nonlinear σ model with twisting are therefore proportional to $\zeta(3)$, in agreement with the low-energy expansion of the four-point tree superstring amplitudes.[9–11]

Finally, a direct calculation of the β function

$$\beta_\lambda \equiv \mu \frac{d}{d\mu} \lambda^2(\mu), \qquad \beta_{ij} = - \frac{\delta_{ij}}{\lambda^4} \beta_\lambda , \tag{7}$$

where μ is a dimensional scale parameter of the renormalization group, with allowance for the single-loop contribution,[3] yields

$$\beta_\lambda = - \frac{-\lambda^4 Q}{2\pi} (1-\eta^2) - \frac{\lambda^{10} Q^4 \zeta(3)}{2^7 (4\pi)^4} (1-\eta^2)[q_0 + q_2\eta^2 + q_4\eta^4 + q_6\eta^6], \tag{8}$$

where Q is an eigenvalue of the second-order Casimir operator in the associated representation of G,

$$f_{imn} f_j^{mn} = Q\delta_{ij} , \tag{9}$$

and the coefficients q_A are

$$q_0 = 18, \qquad q_2 = 94/3, \qquad q_4 = 319, \qquad q_6 = -289 . \tag{10}$$

The Wess-Zumino–Witten term is not renormalized at all; this is what we would have expected on the basis of its topological origin.

The β function of the SWZWM in (8) vanishes at the critical point $\eta^2 = 1$, where the theory exhibits a superconformal symmetry.[4]

[1)]Our conventions and notation are the same as those in Ref. 5.

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