# Resonance tunneling of Cooper pairs in a system of two small Josephson junctions

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The current-voltage characteristics of two, small, series-connected, Josephson tunnel junctions have been calculated. The tunneling of Cooper pairs in these junctions is shown to produce resonant current spikes on the I–V characteristics.

The correlated tunneling of single electrons and Cooper pairs in small tunnel junctions has recently been the subject of active experimental and theoretical research (see, e.g., the review by Likharev[1]). A system consisting of two series-connected junctions, a very simple system from the experimental standpoint, has been studied particularly thoroughly. Only the characteristic features of this system, which are associated with the single-electron tunneling, [1)] have so far, however, been observed experimentally and discussed theoretically.[2–5] The Josephson tunneling was discussed[6] only in the case of low voltage $V$ across the junctions, when the Josephson oscillations occur in the system in accordance with the total Josephson phase difference $\varphi_1 + \varphi_2$. In our study we have considered the region of high voltages $V$, in which the effect of the Josephson tunneling on the dynamics of the system is governed by a diffferent process: the resonant tunneling of Cooper pairs.

The Hamiltonian of this system is the sum of its electrtostatic energy $U(n_1, n_2)$ and the terms $H_{Tj}$ which describe the tunneling through the $j$th tunnel junction[1]:

$$H = U(n_1, n_2) + H_{T1} + H_{T2} \ , \tag{1}$$

$$U(n_1, n_2) = \frac{Q^2}{2C_\Sigma} - \frac{eV}{C_\Sigma}( C_1 n_2 + C_2 n_1) \ , \tag{2a}$$

$$Q = e(n_1 - n_2) + Q_0 \ . \tag{2b}$$

  

Here $C_j$ is the capacitance of the $j$th junction, $en_j$ is the charge that passes through it, $c_\Sigma \equiv c_1 + c_2$,and $Q_0$ is the effective fractional charge (in units of $e$) at the central electrode of the system (which can be induced, for example, by an external electric field). We will restrict the discussion to the case in which the elementary electrostatic energy $E_C = e^2/2C_\Sigma$ is much lower than the characteristic gap values of the energy $|\Delta_j(T) + \Delta_{j'}(T)|$, where $\Delta_j(T)$ are the energy gaps in the superconductors that form the junctions. The tunnel terms $H_{Tj}$ in this case can be represented in the "adiabatic" form

$$H_{Tj} = -E_j \cos\varphi_j + H'_{Tj}, \tag{3}$$

where $H'_{Tj}$ describe the finite below-the-gap quasiparticle conductivity of the junctions, $G_j$.

Assuming that the conductivity $G_j$ and the Josephson binding energy $E_j$ are small $(G_j \ll R_Q^{-1}, R_Q \equiv \pi/2e^2; E_j \ll E_Q)$, we will describe the dynamics of a charge $Q$ at the intermediate electrode by the equation for the density matrix $\rho(Q,Q')$, which was obtained from perturbation theory in $H_{Tj}$. This equation is similar to the equation for a single junction.[7,8] Since the Josephson current is dissipationless, its strongest effect on the dynamics of $Q$ occurs when the difference $\delta(Q)$ in the energies $U$ before and after the tunneling of a Cooper pair through one junction is small, $|\delta| \ll E_Q$. Assuming that the conditions for a resonance in the $j$th junction are satisfied, and retaining only the resonant terms in the equation for $\rho$, we find

$$\begin{aligned} \dot\sigma(Q) &= -E_j \operatorname{Im}[\rho(Q, Q+2e)] + F_T\{\sigma(Q)\}, \\ \dot\sigma(Q+2e) &= E_j \operatorname{Im}[\rho(Q, Q+2e)] + F_T\{\sigma(Q+2e)\}, \\ \dot\rho(Q, Q+2e) &= iE_j[\sigma(Q) - \sigma(Q+2e)]/2 - (i\delta + \nu)\rho(Q, Q+2e), \end{aligned} \tag{4}$$

where

$$\sigma(Q) \equiv \rho(Q, Q), \qquad F_T\{\sigma(Q)\} = \sum_{j,\pm} [\sigma(Q \pm e)\Gamma_j^{\mp}(Q \pm e) - \sigma(Q)\Gamma_j^{\pm}(Q)], \tag{5}$$

$$\nu = \sum_{j,\pm} [\Gamma_j^{\pm}(Q) + \Gamma_j^{\pm}(Q+2e)],$$

and $\Gamma_j^{\prime}$ are the probabilities for single-electron tunneling.[1]

Equations (4) describe, in addition to the single-electron tunneling, the tunneling of Cooper pairs through the $j$th junction with the probability $\gamma_j(Q)$ per unit time:

$$\gamma_j(Q) = \nu E_j^2/2(\delta^2 + \nu^2). \tag{6}$$

The tunneling of Cooper pairs can thus be appreciable only at certain resonance values of $Q$,

$$Q \approx \begin{cases} VC_2 - e, & j = 1, \\ -VC_1 + e, & j = 2; \end{cases} \tag{7}$$

The probability of such tunneling far from these values of $Q$ is small, $\gamma_j \alpha (E_j/E_C)^2$.

Because of the discrete nature of $Q$, the conditions for the resonance can be satisfied only at certain voltages $V$ [at which relations (7) and (2b) are compatible]. As a result, the tunneling of Cooper pairs should lead to the appearance of periodically spaced resonance current spikes on the I-V characteristic of the system. The amplitude of these spikes, however, decreases rapidly with increasing $V$, since at large values of $V$ the probability density $\sigma(Q)$ becomes diffuse and the contribution to the current from a resonant tunneling channel decreases. The I-V characteristic is largely a superposition of such gradually decaying spikes and ordinary single-electron "waves."[1]

The shape of the largest spike (closest to $V=0$) can be easily determined, for example, at zero temperature and at various capacitances of the junctions, $C_1 \ll C_2$. Solving Eqs. (4) for this case, we find that at $G_1 \gg G_2$ the amplitude of the spikes is small, $I_{\max}$ $\alpha G_2/G_1$, and at $G_1 \ll G_2$ the I-V characteristic near the spike $[V \approx (Q_0+e)/C_2)]$ is

$$I = 2e\gamma_1 [\, 1 + 2\gamma_1 \tau \frac{6Q_0 + 5e}{(2Q_0 + 3e)(2Q_0 + e)} ]^{-1}, \qquad \tau \equiv C_\Sigma / G_2 \ , \tag{8a}$$

where

$$\gamma_1 = 2eE_1^2 G_2 C_\Sigma (2Q_0 + 3e) / \{ [8e^2(Q_0 + e - VC_2)]^2 + [G_2(2Q_0 + 3e)]^2 \} \ . \tag{8b}$$

Accordingly, at $E_1 \ll \tau^{-1}$ the amplitude of the spike is $I_{\max} \approx 2eE_1^2\tau$ and its width is $\Delta V \approx (e\tau)^{-1}$ and at $E_1 \gg \tau^{-1} - I_{\max} \approx e/\tau$ we have $\Delta V \approx E_1/e$. At $V<0$ the shape of the I-V curve is determined by the same equations with the replacements $V \to -V$, $I \to -I$, and $Q_0 \to -Q_0$.

The results obtained above make it possible to quantitatively verify whether the current peak observed by Fulton *et al.*[9] can be explained in terms of resonant tunneling of Cooper pairs. The position of this peak with respect to voltage is consistent with that which follows from expressions (2b) and (7). Assuming that below-the-gap conductivity of the junctions (which was not measured by Fulton *et al.*[9]) is proportional to above-the-gap conductivity, we calculate from Eqs. (4) the amplitude of the peak which amounts to 0.6 nA. The deviation from the experimental value, 0.3 nA, is likely due to the nonlinear nature of the below-the-gap conductivity of the junction circuits.

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[1)]After completing this study, we learned that Fulton *et al.*[9] observed a current peak on the I-V characteristics of two Josephson tunnel junction circuits. They explained this peak, correctly in our view, in terms of the resonant tunneling of Cooper pairs. This explanation is briefly discussed at the end of this letter.

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