Internal dynamics of moving domain wall

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A numerical study of the (2 + 1)-dimensional Landau-Lifshitz–Hilbert equation is reported. A new mechanism has been found for the generation and breaching of horizontal Bloch lines.

The equation of motion of the magnetization under the influence of the effective field, i.e., the Landau-Lifshitz–Hilbert (LLH) equation, is fundamental to a study of the dynamics of domain walls. The system of integrodifferential equations with partial derivatives which arises is so complex, however, that the customary approach has been to study average equations: the Slonczewski equations.[1-3] The assumptions which underlie the system of Slonczewski equations are known to be extremely limiting. When the driving field is above a certain critical value, for example, these assumptions do not hold, and all that is possible is a qualitative study of various mechanisms for the dynamics of the domain wall.

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Fast computers have recently made possible substantial progress toward a direct solution of the complete system of LLH equations for calculations on two- and three-dimensional domain-wall structures.[4,5] In these studies, however, the LLH equations have been used as an analog of an iterative method for finding steady-state solutions of static problems. The basic problem in realizing a dynamic version of the numerical method is the absence of a priori information about the nature of the motion of the domain walls.

In the present letter we report the use of an effective algorithm for adaptive variation of the mesh which makes it possible to keep the core of a moving domain wall at the center of the computation region. This is the first use of this algorithm for studying the dynamics of a two-dimensional domain wall.

We consider a ferromagnetic film (a magnetic-bubble material). We assume that the $Z(\mathbf{k})$ axis is the anisotropy axis and is directed perpendicular to the plane of the film, the $X(\mathbf{i})$ axis runs perpendicular to the domain wall and lies in the plane of the film, and the $Y(\mathbf{j})$ axis is directed along the domain wall in such a way that it completes a right-handed coordinate system (Fig. 1). We assume that all the magnetization distributions of interest are translationally invariant along the Y axis. We set $\mathbf{v}(x,z,t) = \mathbf{M}(x,z,t)/M_s, \mathbf{h} = (x,z,t) = \mathbf{H}(x,z,t)/M_S, t_1 = |\gamma|\cdot M_S t, l = (Ak)^{1/2}/(\pi M_S^2)$, and $Q = K/2\pi M_S^2$. Here $\mathbf{M}(x,z,t)$ is the magnetization distribution, $\mathbf{M}_S$ is the saturation magnetization, γ is the gyromagnetic ratio, l is a characteristic length, A is the exchange constant, k is the uniaxial anisotropy constant, $\mathbf{H}(x,z,t)$ is the effective field, and Q is the quality factor. We write the LLH equation in dimensionless form:

$$(1+\alpha^2)\frac{dv}{dt} = \mathbf{h}\times\mathbf{v} - \alpha(\mathbf{v}\times(\mathbf{v}\times\mathbf{h})),$$

$$\mathbf{h} = -4\pi Q\mathbf{v}_\perp + (\pi/Q)\nabla^2\mathbf{v} + \mathbf{h}_{d_1} + \mathbf{h}_{d_2} + \mathbf{h}_0, \tag{1}$$

$$\mathbf{h}_{d_1} = -\int_\Omega \frac{(\nabla\mathbf{v})2\rho}{\rho^2}dxdz + \int_\Omega \frac{(\mathbf{v}\cdot\mathbf{n})2\rho}{\rho^2}ds,$$

where $\mathbf{v} = \mathbf{v}_x i + \mathbf{v}_y j + \mathbf{v}_z k, |\mathbf{v}| = 1, \mathbf{v}_\perp = \mathbf{v}_x i + \mathbf{v}_y j$, ∇ is the Laplacian, $\mathbf{h}_{d_2}$ is the demagnetizing field of neighboring domains, $\mathbf{h}_0$ is the external magnetic field, Ω is the computation region, and α is an attenuation parameter. We write the boundary conditions and the initial condition in the form

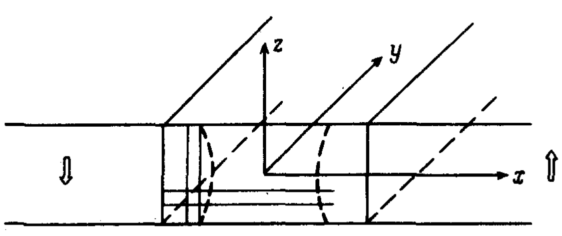

FIG. 1. Geometry of the computation region.

$$\mathbf{v}(\pm L_1, z, t) = \pm k, \qquad \frac{\partial \mathbf{v}}{\partial z}(x, \pm L_2, t) = 0,$$
$$\mathbf{v}(x, z, 0) = \mathbf{v}_0(x, z), \quad (x, z) \in \Omega = [2L_1 \times 2L_2]. \tag{2}$$

System (1)–(2) has been solved by the method of finite differences. As initial conditions we selected the corresponding solution of the static variational problem of the structure of a domain wall without a driving field. The idea of the adaptive algorithm can be summarized as follows: When the driving field $h_0 = \alpha k$, $\alpha > 0$, is applied, the domain wall moves toward the domain with the magnetization direction opposite the field. Clearly, after traveling a certain distance the wall will come to a halt at the edge of the computation region. To keep this event from occurring, we take a "snapshot" of the domain wall after a time interval Δt; if the wall has moved a given distance ΔS, then we displace the mesh along with the domain wall by the corresponding number of nodes. The values of Δt and ΔS are chosen empirically to suit the nature of the motion and the capabilities of the computer.

The results which we are reporting here were calculated for a film with the parameter values $Q = 4$, $D = 3$ (l), $\alpha = 0.2$, and $\mathbf{h}_0 = -4k(M_S)$. The parameters of the computation mesh were $N_x \times N_z = 40$, where N_x is the number of nodes along the x axis, and N_z is that along the z axis.

We denote by $\varphi(x,z)$ the angle between the XY projection of the magnetization vector and the Y axis, and we denote by $\theta(x,z)$ the angle between the magnetization vectors and the z axis. Figure 2(a) shows the familiar process by which a horizontal Bloch line (HBL) is nucleated and breached.[1] The basic process for a twisting angle $\psi(z)$ at the center of the wall $[\psi(z) = \psi(x,z)$ on the line $\theta(x,z) = \pi/2]$ goes as follows.

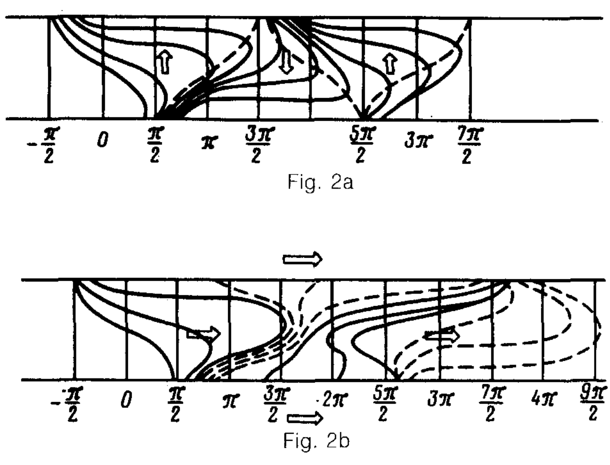

FIG. 2. a—Dynamics of the twisting angle $\psi(z)$ at the center of the domain wall according to the Slonczewski equations; b—the same, according to the solution of system (1)–(2).

1. First, a perturbation of the angle ψ is nucleated at the lower surface of the film. This wave grows and propagates upward. Two HBLs form; a larger one (a 2π HBL) at the top and a smaller one (a π HBL) at the bottom.

2. The upper HBL is breached. At the upper boundary, the vector $\overline{\mathbf{v}}$ rotates through an angle of 4π [not through an angle of 2π, as in Fig. 2(a)].

3. A bit later, a breaching of the lower HBL through an angle 2π begins.

4. The remaining central part of the $\psi(z)$ curve catches up with the boundary values.

A perturbation is then nucleated near the upper boundary, and it grows. The lower 2π HBL is breached; then the upper π HBL is breached; finally, the central region, reaching the edge, completes the cycle.

This dynamics of the angle ψ makes the motion of the individual parts of the domain wall nonuniform. The average velocity over 10 cycles was 0.45 (arbitrary unit l)/(dimensionless time unit t_1).

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