

# The RVB model as the problem of the surface of a quantum crystal

L. S. Levitov

*L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR*

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The dimer RVB model is equivalent to the problem of the surface of a quantum crystal. There are regions on the phase diagram which correspond to a disordered state of the dimers (a quantum liquid) and to an ordered state (a crystal). In the first case, the excitations are gapless, while in the second there is a finite gap.

The dimer RVB model on a square lattice was introduced by Kivelson<sup>1</sup> as the simplest implementation of Anderson's formulation,<sup>2</sup> whose purpose was to find a situation in which the ground state of a quantum system on a lattice is a liquid. In this model, the "states" are all the dense (having no voids) fillings of the edges of the square lattice with dimers (each vertex is covered by precisely one dimer). The simplest Hamiltonian is<sup>3</sup>

$$H = \sum -j(|=\rangle\langle || + \text{H.a.}) + V(|=\rangle\langle =| + ||\rangle\langle ||). \quad (1)$$

The basic problem is that of the structure of the ground state (a crystal or a liquid?) and of the excitations (is there a gap in the spectrum?). To answer these questions, we reduce the dimer problem to another problem, which has been studied more thoroughly.

The following geometric construction proves useful. Each configuration of the dimers is naturally equivalent to an arrangement of  $2 \times 1$  rectangles, i.e., dominoes, in a plane (each rectangle covers one dimer). In turn, each arrangement of dominoes is equivalent to the surface of a 3D crystal. Let us describe the structure of this crystal. We take the polyhedron in Fig. 1: two pup tents resting on the two faces of a square, one being rotated  $90^\circ$  from the other. Working with objects of this type, one can construct layers which, when laid on top of each other, can fill the entire space without voids [Fig. 2(b)]. In neighboring layers, the polyhedra are rotated through  $90^\circ$ . The period of this construction in the vertical direction is four layers. The plane is partitioned into dominoes in a mutually one-to-one correspondence with the surfaces of this crystal (Fig. 2). The surface must be projected without folds onto the basal plane. The triangular faces are projected into line segments.

Any question concerning the dimers can thus be interpreted as a problem concerning the surface of a crystal. Let us pursue this correspondence for the dimer RVB model. The kinetic  $J$  term corresponds to the addition of one polyhedron to the surface and the removal of one polyhedron from the surface; i.e., it describes quantum growth–melting fluctuations of the crystal. The potential  $V$  term represents the energy of the elementary steps. The Hamiltonian of the well-known problem of the surface of a quantum crystal has been obtained.<sup>4</sup> The ordered and liquid states of the dimers can

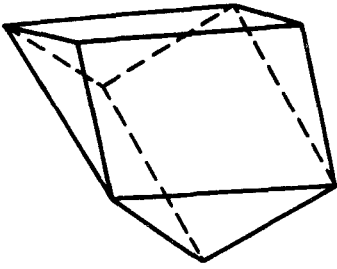


FIG. 1. The polyhedron used to construct the crystal.

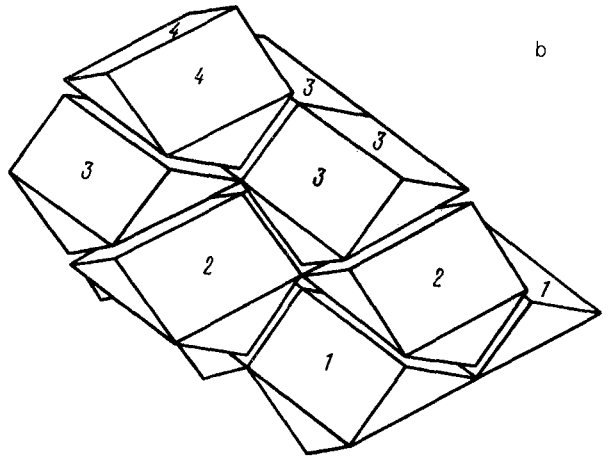
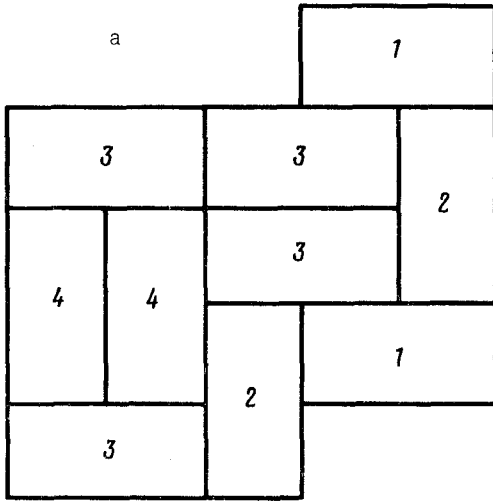


FIG. 2. Correspondence between (a) the configuration of dominoes and (b) the surface of the crystal. The numerals specify the indices of the layers in the crystal.

be identified with smooth and rough states, respectively, of the surface. The low-energy excitations in the dimer problem ("resonons"<sup>3</sup>) correspond to the smooth fluctuations of the crystal surface.

We see that our dimer problem has a hidden macroscopic variable: the average slope of the corresponding surface. The macroscopic nature of the slope, i.e., its slow time dependence, is a consequence of the circumstance that the transformation which has been constructed sends any local Hamiltonian for the dimers into a *local* Hamiltonian for the surface. This degree of freedom has appeared elsewhere and has already been utilized: "topological sectors"<sup>3</sup> and "dipole moment density."<sup>5</sup>

What is the nature of the ground state? We know that the surface of a quantum crystal, having a rational slope, is smooth, regardless of how strong the quantum fluctuations are.<sup>4</sup> Irrational slopes, on the other hand, would correspond to a rough state. In order to find a complete description of the phase diagram, we add to Hamiltonian (1) two terms, which control the average slope:

$$\Sigma \mu_x (|\leftarrow\rangle\langle\leftarrow| - |\rightarrow\rangle\langle\rightarrow|) + \mu_y (|\downarrow\rangle\langle\downarrow| - |\uparrow\rangle\langle\uparrow|). \quad (2)$$

The summation here is over all the dominoes, and the arrows specify the direction of the slope of the corresponding face of the polyhedron.

In the  $(\mu_x, \mu_y)$  plane, to each rational slope there corresponds a region of nonzero area in which this slope is realized. The size of this region is determined in order of magnitude by the energy of the step, which is nonzero in a smooth state. In the case of well-developed quantum fluctuations,  $J \gg V$ , the energy of the step is exponentially small, in accordance with  $J/V$  and in accordance with the Miller indices of the face,<sup>4</sup> so the total area of these regions of the phase diagram is also exponentially small. The energy of the step determines the size of the gap in the spectrum of fluctuations of the surface. Rational slopes of the surface thus correspond to ordered crystalline states of the dimers, and there is a gap in the excitation spectrum.

The remainder of the  $(\mu_x, \mu_y)$  plane, which corresponds to irrational slopes, is of considerably more interest. A rough state with a zero step energy prevails there. This statement means that the dimers form a quantum liquid with gapless excitations. When the quantum fluctuations are large, the rough state of the surface (i.e., the liquid state of the dimers) fills nearly the entire area of the phase diagram.

The dimers corresponding to the four orientations of the arrows in (2) lie on four noninteresting sublattices of the ground lattice. Consequently, the liquid phase can be reached by correctly choosing the chemical potentials on these sublattices.

We note also that in the case of strong quantum fluctuations the correlation radius  $r_c$  at which the difference between the rational and irrational slopes begins to be seen is exponentially large, in accordance with  $J/V$ .<sup>4</sup> In this case, the system of dimers behaves as a liquid over scales smaller than  $r_c$  for all  $\mu_x, \mu_y$ .

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<sup>4</sup>D. S. Fisher and J. D. Weeks, *Phys. Rev. Lett.* **50**, 1077 (1983); S. V. Iordanskiĭ and S. E. Korshunov, *Zh. Eksp. Teor. Fiz.* **87**, 927 (1984) [*Sov. Phys. JETP* **60**, 1617 (1984)]; E. Fradkin, *Phys. Rev. B* **28**, 5338 (1983).

<sup>5</sup>L. B. Ioffe and A. I. Larkin, *Phys. Rev. B*, 1989, in press.

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