Supersymmetric coset models in terms of free superfields

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It is proposed that explicitly $N=1$ supersymmetric $N=2$ Kazama and Suzuki models be constructed in terms of free $N=1$ superfields through an appropriate $N=1$ generalization of the Wakimoto bosonization. Explicit constructions have been carried out for a super-$SL(2)/U(1)$ coset theory. As a result, minimal $N=2$ models are generated.

The basic problem of string theory—the description of four-dimensional physics—is solved through a compactification on six-dimensional Calabi–Yau manifolds. The discovery of an equivalent description of these compactifications in terms of two-dimensional $N=2$ superconformal theories has turned out to be very important.[1] In addition to the minimal $N=2$ models,[2,3] a significant number of new $N=2$ theories corresponding to Calabi–Yao manifolds were found in Ref. 4. In the present letter we wish to call attention to the possibility of a description of these theories in terms of free superfields, which follows from the use of an appropriate generalization of the Wakimoto–Feĭgin–Frenkel'–Zamolodchikov–Dotsenko-... bosonization (we will say "Wakimoto bosonization" for brevity) of the Wess–Zumino–Witten models and related theories.[5–12] Below we construct a Wakimoto superbosonization in the simplest case, $SL(2)$, but it can easily be constructed for arbitrary (semi-) simple Lie algebras in the spirit of Refs. 9–11, which dealt with the $N=0$ case. We find an $N=1$ superfield description—precisely what we need for an $N=1$ generalization[4] of the GKO coset construction.[13] A description in terms of *free* $N=1$ superfields makes it possible to explicitly determine whether any $N=1$ coset model is actually ($N=2$)-supersymmetric: The so-called $U(1)$-(super)current which is required for $N=2$ supersymmetry follows in a natural way from current algebra, and an explicit test of the necessary operator expansions can easily be carried out in a free theory. This procedure should therefore provide an independent derivation of the condition which will tell us whether a given $N=1$ coset model is actually ($N=2$)-supersymmetric (Ref. 4; see also Ref. 14).

A program of this sort is developed below for the very simple example of a super-$SL(2)/U(1)$ construction, which leads to minimal $N=2$ models.

We recall that the ordinary Wakimoto bosonization for $SL(2)$ is constructed with the help of a pair of commuting first-order fields (the $\beta\gamma$ system, with spin 1) and an independent scalar field ψ. Our first step is analogous to superbosonization for the $N=1$ Wess–Zumino–Witten $SL(2)$ theory, which is described by three fermion superfields of conformal dimensionality 1/2 with the operator products[15–17]

$$J^0(1)J^{\pm}(2)=2\frac{\theta_{12}}{z_{12}}J^{\pm},\quad J^+(1)J^-(2)=-\frac{k}{z_{12}}-\frac{\theta_{12}}{z_{12}}J^0,$$

$$J^0(1)J^0(2)=2\frac{k}{z_{12}},\qquad (1a, b, c)$$

where, as usual, $\theta_{12}=\theta_1-\theta_2$, $z_{12}=z_1-z_2-\theta_1\theta_2$ and below we will also use $D=\partial/\partial\theta+\theta\partial/\partial z$, $\partial=\partial/\partial z$.

In order to bosonize the three "currents" $J^{\pm}$, J^0, we use a combined system of commuting and anticommuting order field, $\beta\gamma$ and bc, which constitute two superfields B and C of conformal dimensionalities 1/2 and 0, respectively[1]:

$$B=b+\theta\beta,\quad C=\gamma+\theta c\quad B(1)C(2)=-\theta_{12}/z_{12}\qquad (2)$$

We now assume, by analogy with the $N=0$ case,

$$J^+=B,\quad J^0=\sqrt{2k}\partial\Psi+2BC,\quad J^-=BC^2+\sqrt{2k}CD\Psi+kDC,\qquad (3a, b, c)$$

where Ψ is a scalar superfield with an ordinary operator product

$$D\Psi(1)D\Psi(2)=\frac{1}{z_{12}}.\qquad (4)$$

The super-$SL(2)$ operator products [i.e., the singularities in (1)] which we need are then indeed reproduced. In addition to the singularities, however, we will need the finite parts of the operator products, i.e., the composite operators of the type $:J^aJ^b:$, expressed in terms of the fields of the BC-Ψ theory. In (1b), for example, we find

$$J^+(1)J^-(2)=-\frac{k}{z_{12}}-\frac{\theta_{12}}{z_{12}}J^0+k\mathcal{H},\quad \mathcal{H}=BDC+\sqrt{\frac{2}{k}}BCD\Psi.\qquad (5)$$

For the energy-momentum supertensor (here and below, in the Neveu–Schwarz sector)[17,20] we have

$$T=-\frac{1}{2k}:(DJ^a)J^a:+\frac{1}{6}f^{abc}:J^a:J^bJ^c::$$

$$=-\frac{1}{2}DBDC-\frac{1}{2}B\partial C+\frac{1}{2}\partial\Psi D\Psi-\frac{1}{\sqrt{2k}}\partial D\Psi.\qquad (6)$$

The first two terms on the right side constitute the energy-momentum supertensor in the BC theory [see the equation on page 252 in Ref. 18, with a change in the field designations: $B\leftrightarrow C$ (see also Ref. 19)]. The BDC term in (5) is an $N=2$ $U(1)$ supercurrent in the BC theory.[18,19]

The BC system can in turn be "superbosonized"[18,19]:

$$B=-e^{-\phi}D\bar{\phi},\quad C=e^{\phi},\quad \phi(1)\phi(2)=\log z_{12},\qquad (7)$$

where ϕ and $\bar{\phi}$ are therefore scalar superfields (with nonsingular $\phi\phi$ and $\bar{\phi}\bar{\phi}$). We then find the following result for the $SL(2)$ supercurrents:

$$J^+=-D\bar{\phi}e^{-\phi},\ J^0=\sqrt{2k}\partial\Psi-2D\bar{\phi},\ J^-=(\sqrt{2k}D\Psi+kD\phi-D\bar{\phi})e^{\phi}. \tag{8a, b, c}$$

These and similar expressions follow from the basic operator expansion

$$B(1)C(2)=-\frac{\theta_{12}}{z_{12}}-D\bar{\phi}(2)+\theta_{12}(-\partial\bar{\phi}(2)+\partial\phi(2)+D\phi(2)D\bar{\phi}(2))$$
$$+z_{12}(\partial\phi(2)D\bar{\phi}(2)-\partial D\phi(2)) \tag{9}$$

(plus higher-order terms). In particular, the energy-momentum supertensor becomes

$$T=\frac{1}{2}D\phi\partial\bar{\phi}+\frac{1}{2}\partial\phi D\bar{\phi}-\frac{1}{2}\partial D\phi+\frac{1}{2}\partial\Psi D\Psi-\frac{1}{\sqrt{2k}}\partial D\Psi. \tag{10}$$

We can also write expressions for the "primary" fields of a super-$SL(2)$ Wess–Zumino–Witten model in terms of vertex operators:

$$V_{j,m}=e^{\sqrt{\frac{2}{k}}j\Psi}e^{(j-m)\phi} \tag{11}$$

where[2)] $m=-j,\ -j+1,\ldots,j$. These fields satisfy the coalescence rules

$$J^{\pm}(1)V_{j,m}(2)=\mp(j\mp m)\frac{\theta_{12}}{z_{12}}V_{j,m\pm1},\quad J^0(1)V_{j,m}(2)=2m\frac{\theta_{12}}{z_{12}}V_{j,m}.$$

(There is also an "adjoint" set of vertex operators, which represent a Wakimoto comodulus, but we will not discuss them here.)

The next step in the construction of a super-$SL(2)/U(1)$ coset model is to single out the $U(1)$ factor in the operators written above. This can be done quite easily by introducing a new basis in the space of the three scalars

$$DV=D\bar{\phi},\ DU=D\Psi+\sqrt{\frac{2}{k}}BC=D\Psi-\sqrt{\frac{2}{k}}D\bar{\phi}, \tag{12a, b}$$

$$DW=\frac{1}{k}BC+\sqrt{\frac{2}{k}}D\Psi+D\log C=\sqrt{\frac{2}{k}}D\Psi+D\phi-\frac{1}{k}D\bar{\phi}, \tag{12c}$$

where DU is essentially J^0, while the two other currents, DV and DW, are orthogonal to DU in the operator-product sense. In other words, they have no singularities in a coalescence with DU. We find

$$DU(1)DU(2)=\frac{1}{z_{12}},\quad DV(1)DW(2)=\frac{1}{z_{12}} \tag{13}$$

with nonsingular $DVDV$ and $DWDW$.

The energy-momentum supertensor now splits up into $U(1)$ and $SL(2)/U(1)$ parts,

$$T=\frac{1}{2}DW\partial V+\frac{1}{2}DV\partial W-\frac{1}{2}D\partial W-\frac{1}{2k}\partial DV+\frac{1}{2}DU\partial U\equiv T_{min}+\frac{1}{2}DU\partial U, \tag{14}$$

and an explicit calculation shows that the energy-momentum tensor of the coset model, T_{min}, has the operator expansion

$$T_{min}(1)T_{min}(2)=\frac{k-2}{2k}\,\frac{1}{z_{12}^3}+\ldots \tag{15}$$

In other words, we have obtained a minimal $N=2$ model with a central charge $c=(3\hat{k}/(\hat{k}+2)$, where[3)] $\hat{k}=k-2$.

In the $N=2$ model there must exist a supercurrent $\mathscr{H}_{min}$ with the properties

$$\mathcal{H}_{min}(1)\mathcal{H}_{min}(2)=\frac{c/3}{z_{12}^2}+\frac{\theta_{12}}{z_{12}}2T_{min} \tag{16}$$

$$T_{min}(1)\mathcal{H}_{min}(2)=\frac{\theta_{12}}{z_{12}^2}\mathcal{H}_{min}(2)+\frac{1}{2}\,\frac{1}{z_{12}}D\mathcal{H}_{min}(2)+\frac{\theta_{12}}{z_{12}}\partial\mathcal{H}_{min}(2) \tag{17}$$

We have already noted that the first term in (5) has similar properties in the BC theory. The supercurrent $\mathscr{H}$ does indeed give us the $\mathscr{H}_{min}$ which we need, after the $U(1)$ part is separated out:

$$\mathcal{H}=\partial\phi+D\phi D\bar{\phi}+\sqrt{\frac{2}{k}}\,D\Psi D\bar{\Psi}=\partial W-\frac{1}{k}\partial V+DWDV-\sqrt{\frac{2}{k}}\,\partial U\equiv\mathcal{H}_{min}-\sqrt{\frac{2}{k}}\,\partial U. \tag{18}$$

Operator expansions (16) and (17) can now be verified easily.

Singling out the $U(1)$ component in the vertex operators, we find

$$V_{j,m}=e^{m\sqrt{\frac{2}{k}}U}M_{j,m},\quad M_{j,m}=e^{\frac{j+m}{k}V}e^{(j-m)W} \tag{19}$$

The vertex operators $M_{j,m}$, which are primary fields of the minimal model, are characterized by their own dimensionalities and by the $\mathscr{H}_{min}$ charges[4)] found from the expansions

$$\mathcal{H}_{min}(1)M_{j,m}(2)=\frac{2m}{k}\,\frac{1}{z_{12}}M_{j,m}(2)+\frac{\theta_{12}}{z_{12}}DM_{j,m}(2) \tag{20}$$

$$T_{min}(1)M_{j,m}(2)=\frac{j(j+1)-m^2}{k}\,\frac{\theta_{12}}{z_{12}^2}M_{j,m}(2)+\frac{1}{2}\,\frac{1}{z_{12}}DM_{j,m}(2)+\frac{\theta_{12}}{z_{12}}\partial M_{j,m}(2). \tag{21}$$

Here, however, j (and therefore m) takes on integer and *half-integer* values. We introduce $l=2j$, $q=2m$, so we can work with exclusively integers. We then find vertex operators whose dimensionalities and $U(1)$ charges are (we recall that $\hat{k}=k-2$)

$$M(l,q)=e^{\frac{l+q}{2k}V}e^{\frac{l-q}{2}W},\quad \Delta(l,q)=\frac{l(l+2)-q^2}{4(\hat{k}+2)},\quad Q(l,q)=\frac{q}{(\hat{k}+2)},\quad q=-l,-l+2,\ldots,l, \tag{22}$$

the same as the known values in the Neveu–Schwarz sector[5)] (Refs. 2, 3, and 22). Note also that there are some other representatives of primary fields with the same dimensionalities and $U(1)$ charges:

$$\widetilde{M}(l,q)=e^{\frac{q-l-2}{2k}V}e^{-\frac{l+q+2}{2}W}. \qquad (23)$$

These $\widetilde{M}(l,q)$ must be used along with the operators $M(l,q)$ in the construction of correlation functions, as in Refs. 23, 7, 8, 6, and 9. The correlation functions can now be expressed easily (for a four-tail, say) in terms of entities which could naturally be called "superhypergeometric functions." It is to be understood, of course, that the necessary insertions of screening operators in the correlation functions are carried out.[23,7,8]

The Ramond sector of minimal $N=2$ models, an explicit construction of correlation functions, and applications to the Kazama and Suzuki theories themselves will be discussed in a more detailed paper.

[1)] Since the dimensionality of the $\beta\gamma$ system is larger by 1/2 than that of the bc system in our case, while it is smaller by 1/2 in the standard description, the structure of the superfields in (2) is nonstandard.

[2)] The presence of the constraint[22] $j\leqslant\hat{k}/2$ is not visible in the bosonized version. This circumstance may indicate that the description is somewhat incomplete. The $N=0$ case also suffers from the absence of an explicit constraint on j.

[3)] The normalization of the central charge corresponds to an $N=0$ description, so we have $T(1)T(2)=(c/6)(1/z_{12}^3+...$ for the energy-momentum supertensor.

[4)] Now that the $U(1)$ factor has been eliminated from SL(2)/U(1), we can, without any fear of confusion, call the supercurrent $\mathscr{H}_{min}$ a "$U(1)$ current" and the $\mathscr{H}_{min}$ charge a "$U(1)$ charge," as in $N=2$ supersymmetry.

[5)] As before,[2)] there is no constraint $l\leqslant k$; in fact, there is not even any indication that k is an integer (at this point we are not attempting to derive correlation functions).

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