Maser generation of curvature radiation

V. S. Beskin, A. V. Gurevich, and Ya. N. Istomin
USSR, Moscow

(Submitted 31 October 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **50**, No. 11, 450–452 (10 December 1989)

There is the possibility of experimentally observing a new method for generating electromagnetic waves in the millimeter and submillimeter ranges.

Electrons of a relativistic energy $mc^2\gamma$ moving along a field line of a curvilinear magnetic field radiate electromagnetic waves. This "curvature radiation" is very similar to ordinary synchrotron radiation,[1] having a characteristic frequency $\omega \approx \frac{c}{\rho}\gamma^3$, where ρ is the radius of curvature of the magnetic field. The radiation is concentrated in a narrow angle $\theta \approx 1/\gamma$ along the direction of the magnetic field. The wavelength of the curvature radiation with $\rho \sim 10^2$ cm and $\gamma \sim 10^1$–10^2 cm. If the radiation is generated by a sufficiently dense beam of electrons, for which the distance between particles is much smaller than the characteristic wavelength $\lambda \gg n^{-1/3}$, the generation of curvature radiation should be of a collective maser nature. This situation can be arranged in existing intense relativistic electron beams, with values $\gamma \gtrsim 10$ and total currents reaching $I \gtrsim 1$ kA.

A particularly strong maser effect occurs in a dense plasma, in which electromagnetic modes having a refractive index greater than unity can propagate, so the stage is set for the simultaneous operation of the curvature and Cerenkov radiation mechanisms. The joint operation and interference of the synchrotron and Cerenkov mechanisms have been observed previously in the case of spontaneous emission in a low-current electron beam passing through a gas of finite density.[2]

Let us examine the specific conditions for the maser emission of curvature radiation. According to the theory of Ref. 3, maser amplification is possible if the charged-particle density n in the beam is sufficiently high:

$$4\pi \frac{\omega_p^2 \rho^{4/3}}{\omega^{2/3} c^{4/3} \gamma^3} > 1. \tag{1}$$

Under condition (1), the growth rate of the unstable oscillations, Γ, is of a hydrodynamic nature and is given by

$$\Gamma = \omega^{1/5} \omega_p^{2/5} (c/\rho)^{2/5} \gamma^{-3/5} . \tag{2}$$

As in the case of synchrotron radiation, the unstable waves propagate in a narrow angular cone $\theta < \theta_\parallel$ along the direction in which the relative particles are moving; here

$$\theta_\parallel = \omega_p^{4/5} \rho^{1/5} \omega^{-3/5} c^{-1/5} \gamma^{-6/5} . \tag{3}$$

The curvature of the magnetic field causes the wave vector to deviate from the direction of the beam. As a result, the wave moves out of the amplification region. The gain $\Gamma\rho\theta_\parallel/c$ thus remains finite. If a significant maser effect is to be observed, the gain must be at least on the order of 10; i.e.,

$$\omega^{-2/5} \omega_p^{6/5} \gamma^{-9/5} (c/\rho)^{-4/5} \gtrsim 10 . \tag{4}$$

Condition (4) is essentially the same as (1) but more stringent. Another condition which must be satisfied if there is to be a strong hydrodynamic instability with growth rate (2) is that the possible spread in electron velocities in the beam are small: $\Delta v_\parallel / c < \Gamma/\omega$. Since we have $v_\parallel = c(1 - 1/2\gamma^2)$, we also have $\Delta v_\parallel / c = (1/\gamma^2)(\langle\Delta\gamma\rangle/\gamma)$, where $\langle\Delta\gamma\rangle$ is the average energy spread of the particles. As a result, we find

$$\omega_p^{2/5} \omega^{-4/5} (c/\rho)^{2/5} \gamma^{7/5} > \langle\Delta\gamma\rangle/\gamma . \tag{5}$$

The two conditions in (4) and (5) determine the region of parameter values for which the maser amplification of radiation can be observed. From (4) and (5) follow the inequalities

$$\rho/\gamma^{9/4} > 20 c \omega^{1/2} \omega_p^{-3/2} ;$$

$$\rho/\gamma^{7/2} < c \omega_p \omega^{-2} (\langle\Delta\gamma\rangle/\gamma)^{-5/2} , \tag{6}$$

whose simultaneous satisfaction determines the characteristic frequency of the waves being amplified:

$$\omega \lesssim 0.3\, \omega_p \gamma^{1/2} (\langle\Delta\gamma\rangle/\gamma)^{-1} . \tag{7}$$

It is convenient to express everything in terms of the basic parameters of the beam of relativistic particles: the total current I, the beam radius R, and the electron energy γ. Since we have

$$\omega_p = 2.6\times 10^{10} I_{\text{kA}}^{1/2}/R\ s^{-1},$$

where I_{kA} is the current in kiloamperes, and R is in centimeters, condition (7) can be rewritten as

$$\omega \lesssim 0.8\times 10^{10}(I_{\text{kA}}\gamma)^{1/2}R^{-1}(\langle\Delta\gamma\rangle/\gamma)^{-1}. \qquad (8)$$

The optimum radius of curvature also follows from (6):

$$\rho \approx 13\gamma^{5/2} R I_{\text{kA}}^{-1/2}(\langle\Delta\gamma\rangle/\gamma)^{-1/2}. \qquad (9)$$

The wavelength of the radiation which is generated must be small in comparison with the beam radius:

$$\lambda = \frac{2\pi c}{\omega} \ll R.$$

This condition, along with (8), leads to

$$I_{\text{kA}}\gamma \gtrsim 6\times 10^3(\langle\Delta\gamma\rangle/\gamma)^2. \qquad (10)$$

Another condition which must be satisfied if a Cerenkov resonance is to be arranged is that the curved magnetic field be sufficiently strong: $\omega_B/\gamma \gg \Gamma$. Using (2), we see that this condition means that the following must hold:

$$B_{\text{kG}} > 2\rho_{\text{cm}}^{-2/5}\gamma^{9/10}(I_{\text{kA}}/R^2)^{3/10}. \qquad (11)$$

Under conditions (10) and (11) it is thus possible to observe a collective curvature emission of electromagnetic waves in frequency region (8). According to the present understanding, it is curvature maser radiation which is the basis for the intense radio emission of pulsars.[4] In other words, the effect pointed out here is apparently observed in astrophysics.

We wish to thank V. L. Ginzburg and A. A. Rukhadze for a useful discussion.

[1]L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Addison–Wesley, Reading, Mass., 1971.
[2]K. D. Bonin *et al.*, Phys. Rev. Lett. **57**, 2264 (1986).
[3]V. S. Beskin *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 18 (1986) [JETP Lett. **44**, 20 (1986)].
[4]V. S. Beskin *et al.*, Astrophys. Space Sci. **146**, 205 (1988).

Translated by Dave Parsons