Anomalous damping of low-frequency edge magnetoplasma oscillations in case of quantum Hall effect

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The diffusion component of the current near a boundary gives rise to an additional damping of low-frequency edge magnetoplasma oscillations in inhomogeneous 2D electron systems (including superlattices). This damping increases with decreasing value of the dissipative conductivity σ_{xx}. The possibility of experimentally observing the anomalous damping of edge magnetoplasma oscillations in the case of an integer quantization of the Hall effect is discussed.

In the absence of an external magnetic field, plasmons in 2D electron systems[1] are damped only in the collisionless limit $\omega\tau \gg 1$, where τ is the momentum relaxation time. In a strong magnetic field, such that the condition $\omega_c\tau \gg 1$ holds (ω_c is the cyclotron frequency), and Hall currents outweigh the dissipative currents, weakly damped low-frequency magnetoplasma oscillations can exist in both the collisionless limit and the local hydrodynamic limit $\omega\tau \ll 1$. Weakly damped magnetoplasma oscillations with frequencies below ω_c and $1/\tau$ have recently been observed experimentally and are presently the subject of active research. These are edge magnetoplasma oscillations (EMOs) in bounded 2D electron systems under conditions corresponding to the quantum Hall effect (see, for example, Refs. 2–5 and the bibliographies there). The EMO frequencies are inversely proportional to the magnetic field and to the transverse dimension of the 2D system. It follows from recent experiments[6] that the damping of low-frequency ($\omega\tau \ll 1$) EMOs is not described at all by the existing theories. In the present letter we show that if the ratio of the dissipative and Hall conductivities of inhomogeneous 2D systems (including superlattices) is sufficiently small, $\sigma_{xx}/\sigma_{xy} \ll 1$, the diffusion component of the electric field near a boundary leads to a damping of EMOs which increases with decreasing value of the ratio σ_{xx}/σ_{xy}. Under conditions corresponding to the quantum Hall effect in strong magnetic fields, this mechanism may lead to an anomalous damping and to the disappearance of the EMOs. This property of EMOs stems from the vanishing of the normal component of the current at the boundary of the system; in the case of a vanishingly low dissipative conductivity, while the Hall conductivity is nonzero, the result will be a pronounced increase in the gradients of the nonequilibrium-carrier density near a boundary and an increase in the dissipation. In an ideal Hall sample ($\sigma_{xx} = 0$, $\sigma_{xy} \neq 0$), the boundaries of the sample are equipotentials, so edge magnetoplasma waves accompanied by oscillations of the electron density and the boundary potential cannot propagate.

1. We consider a semi-infinite ($y < 0$) conducting 3D medium in an external magentic field $H_0 \| Z$. Effectively qualifying as a medium of this sort might be, for example, a superlattice of 2D electron layers separated by thin insulating interlayers of thickness d in the limit $kd \ll 1$ (Refs. 7 and 8). This effect can be described correctly only if the drift and diffusion components of the current are taken into account simultaneously in the magnetized conducting medium, so the system of equations for low-frequency oscillations ($\omega\tau \ll 1$) consists of the Poisson equation, the equations of electrostatics, and the charge conservation law, along with the constitutive equations for the electric displacement $\mathbf{D} = \epsilon \mathbf{E}$ and the current density $\mathbf{j} = \hat{\sigma}(\mathbf{E} - \beta \nabla \rho)$, where ϵ, ρ, and $\hat{\sigma}$ are respectively the dielectric constant, volume charge density, and conductivity tensor of the 3D medium, and $\beta > 0$ is the Einstein coefficient. At the $y = 0$ interface between a conducting medium and an insulating external medium (with a dielectric constant ϵ_0), boundary conditions are imposed: The electostatic potential φ and the normal component of the electric displacement, D_y, are continuous, and the normal component of the current, j_y, vanishes. For the frequency $\omega(k)$ of a surface magnetoplasmon which is propagating along this boundary, in the direction across the external magnetic field (Refs. 9 and 10, for example), we find the dispersion relation

$$(\sigma_{xx} + i\sigma_{xy})(\epsilon \kappa_D + \epsilon_0 k) = \frac{i\omega\,\epsilon}{4\pi\sigma_{xx}}(\epsilon + \epsilon_0)(\sigma_{xx}\kappa_D + ik\sigma_{xy}), \tag{1}$$

where the parameter $\varkappa_D = [k^2 + (4\pi\sigma_{xx}/\epsilon - i\omega)/\beta\sigma_{xx}]^{1/2}$ is the reciprocal of the depth (δ) to which oscillations in the carrier density penetrate into the conducting medium ($1/\delta = \operatorname{Re} \varkappa_D$). In the case of a small but nonzero σ_{xx}, corresponding to the conditions $\sigma_{xx} \ll \sigma_{xy}$, $k\delta \ll \sigma_{xx}/\sigma_{xy}$, in which case we have $\delta = (2\beta\sigma_{xx}/\omega)^{1/2}$, the spectrum ω' and the damping ω'' of a surface magnetoplasmon take the form

$$\omega' = \frac{4\pi\sigma_{xy}}{\epsilon + \epsilon_0}, \qquad \omega'' = \frac{4\pi}{\epsilon + \epsilon_0}\left(\sigma_{xx} + \frac{\sigma_{xy}^2}{\sigma_{xx}} k\delta\right), \tag{2}$$

In other words, in the case $(\sigma_{xx}/\sigma_{xy})^2 \ll k\delta \ll \sigma_{xx}/\sigma_{xy}$ the damping of a surface magnetoplasmon is determined completely by the diffusion processes near the boundary, and it does indeed increase with decreasing value of the ratio σ_{xx}/σ_{xy}, as $\omega'' \propto (\sigma_{xy}/\sigma_{xx})^{1/2}$. In the case of vanishingly small values of σ_{xx} $(\sigma_{xx}/\sigma_{xy} \ll k\delta \ll 1)$, on the other hand, the quality factor of surface magnetoplasma oscillations becomes less than unity, and these weakly damped surface magnetoplasmons cease to propagate.

The spectrum and damping of surface magnetoplasmons of the type in (2) in the region of their weak absorption can be found not only from dispersion relation (1) but also directly through the use of the surface-charge conservation law $p_S = -i\omega'\rho_S = -j_y = -ik\varphi_S\sigma_{xy}$, $\pi\rho_S = k\varphi_S(\varepsilon + \varepsilon_0)$, the dissipation function ψ $(\omega'' = \psi/2\mathscr{E})$,

$$\psi = \int_V dV\sigma'_{xx}\left[|E_x - \beta\frac{\partial\rho}{\partial x}|^2 + |E_y - \beta\frac{\partial\rho}{\partial y}|^2\right] \tag{3}$$

and the electrostatic energy of the oscillations, $\mathscr{E}$,

$$\mathscr{E} = \int_V dV\frac{\epsilon}{8\pi}(|E_x|^2 + |E_y|^2) + \int_{V_0} dV_0\frac{\epsilon_0}{8\pi}(|E_x|^2 + |E_y|^2). \tag{4}$$

The vanishing of the total current inside the diffusion layer near the interface is taken into account here: $ik\varphi_S\sigma_{xy} - \sigma_{xx}\beta\rho/\delta \approx 0$. In expressions (3) and (4), V and V_0 are the regions of the conducting and external media. The integral outside (inside) the diffusion surface layer determines the first (second) term in expression (2) for the damping of the surface magnetoplasmons.

2. A similar qualitative method can be used to find the spectrum and damping of EMOs (in the region of their weak absorption), with allowance for the inhomogeneity of the carriers near the boundary in a bounded 2D electron system (see also Ref. 4). Working from the law of conservation of the edge charge Q, which sets up a potential $\varphi = 2Q\ln(1/kr)\exp(ikx - i\omega t)$ in vacuum (r is the distance from the edge of the 2D system, k is the wave number along it, and $kr \ll 1$), and the expressions for the dissipation function and the electrostatic energy, as in expressions (3) and (4), we find the following logarithmic-accuracy estimate of the spectrum and damping of the EMOs in the case $k\delta \ll \sigma_{xx}/\sigma_{xy}$, $k\delta \ll 1$:

$$\omega'_p = 2\sigma_{xy}k\ln(1/k\delta),$$

$$\omega''_p = 2\sigma'_{xx}\left[\frac{1}{\delta\ln(1/k\delta)} + \frac{k}{\ln(1/k\delta)} + |\frac{\sigma_{xy}}{\sigma_{xx}}|^2 k^2\delta\ln(1/k\delta)\right], \tag{5}$$

$$\omega'_p \gg \omega''_p,$$

For the depth (δ) of the localization of the nonequilibrium carriers near the edge of the 2D system, the following expression holds in our diffusion approximation:

$$\frac{1}{\delta} = \mathrm{Im}\left\{\left[\left(\frac{\pi}{\beta}\right)^2 + \frac{i\omega}{\beta\sigma_{xx}}\right]^{1/2} - \frac{\pi}{\beta}\right\}. \tag{6}$$

The first and second terms in (5) for ω'' of the EMOs are determined by the dissipation outside layer δ, while the third is determined by the dissipation in the layer itself. At frequencies $\omega \ll |\sigma_{xx}|/\beta$, we find from (5) and (6)

$$\delta = 2\pi |\sigma_{xx}|^2 / (\omega \sigma'_{xx}), \quad \omega'' \approx 2\pi\sigma_{xy} k + 2\sigma'_{xx} k / \ln(1/k\delta). \tag{7}$$

Expression (7) for ω'' of the EMOs differs from the corresponding expression which has been derived by Volkov and Mikhaĭlov[4] in that it contains a term proportional to $\sigma'_{xx} k$. This term can contribute significantly to the damping of EMOs in magnetic fields (or of the value of ε_F of the 2D electrons) which do not correspond to the middle of the plateau of quantized values of σ_{xy}, in which case σ'_{xx} depends strongly on the external parameters and increases significantly.[6] At frequencies $\omega \gg |\sigma_{xx}|/\beta$, the depth δ is equal to the diffusion length $[\delta = (2\beta\sigma_{xx}/\omega)^{1/2} \gg 2\pi\sigma_{xx}/\omega]$, and it follows from (5) that under the condition $\sigma_{xx}/[\sigma_{xy} \ln(1/k\delta)] \ll k\delta \ll \sigma_{xx}/\sigma_{xy}$ the damping of the EMOs is determined completely by the diffusion currents near the boundary. It increases with decreasing value of the ratio σ_{xx}/σ_{xy}, as $\omega'' \propto (\sigma_{xy}/\sigma_{xx})^{1/2}$ [if we ignore $\sigma''_{xx}(\omega)$ in the limit $\omega\tau \ll 1$; cf. expression (2) for the damping of surface magnetoplasmons]. In the case $k\delta \gtrsim \sigma_{xx}/\sigma_{xy}$, in contrast, the EMOs become strongly damped (with a quality factor less than unity). The transition from weak damping to anomalous damping of EMOs thus occurs at $\omega \sim \sigma_{xx}/\beta$, $\delta \sim \beta$, i.e., under the condition

$$\sigma_{xx}/\sigma_{xy} \lesssim k\beta \ln(1/k\beta), \tag{8}$$

where we have $\omega''/\omega' \sim \sigma_{xy} k\delta/\sigma_{xx} \sim 1/\ln(1/k\beta) \ll 1$.

Working from experimental data on the density of states near the Fermi level, $D(\varepsilon_F)$, for energies ε_F between Landau levels,[11] we find the estimate $\beta = 1/e^2 D(\varepsilon_F) \sim 10^{-5}$ cm. In strong magnetic fields $H_0 \approx 10$T we thus have $\delta \sim \beta \gg r_c \sim v_F/\omega_c \sim 10^{-6}$ cm and $\delta \gg \ll \alpha_H = (\hbar c/eH_0)^{1/2} \sim 10^{-6}$ cm, justifying our use of the local hydrodynamic approximation under these conditions (r_c is the radius of a cyclotron orbit, and α_H is the magnetic length).[12,13] For the values $k \sim 10$ cm^{-1} in the experiments of Ref. 6, we find from (8) a lower limit on the values of the dynamic dissipative conductivity $\sigma_{xx}(\omega_p)$ of a sample at which there is an anomalous damping of EMOs:

$$\sigma_{xx}/\sigma_{xy} \sim 10^{-3} - 10^{-4}.$$

A dynamic conductivity of this magnitude could apparently be achieved in high-quality 2D conducting channels with a high carrier mobility in strong magnetic fields (for small integer values of the filling factor, $\nu = 1,2$).

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