Increase in superfluid transition temperature in polarized Fermi gas with repulsion

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A magnetic field can intensify the Kohn-Luttinger effect, i.e., can lead to a substantial increase in the temperature of P pairing in a low-density Fermi gas with repulsion. The temperature T_c goes through a maximum at a degree of polarization $\alpha = 0.48$. The two-dimensional case has also been studied.

1. In 1965, Kohn and Luttinger[1] showed that the long-range component of the effective potential of the interaction of particles through the Fermi background causes a Fermi system with repulsion to be definitely unstable with respect to a transition to a superfluid state with a large orbital angular momentum in the relative motion of a Cooper pair. We have recently shown[2] that in a low-density Fermi gas ($ap_F \ll 1$, where a is the s-scattering length) this effect persists to the value $l = 1$, and a calculation shows that the gas is unstable with respect to P pairing (the corresponding harmonic of the effective interaction reaches a maximum in absolute value).

Our purpose in the present letter is to show that the critical temperature varies in a nonmonotonic way with the degree of polarization α, reaching a maximum at $\alpha = 0.48$.

The physical reason for the increase in T_c in a magnetic field can be seen as we go to large orbital angular momenta $l \gg 1$. This physical reason is that as the Fermi spheres for the components with different projections move apart, the Fourier component of the effective interaction $\widetilde{\Gamma}(\vartheta)$ becomes a nonanalytic function of ϑ, the angle between the incoming and outgoing momenta, at a value of ϑ different from π (this nonanalytic behavior reflects the oscillatory nature of the interaction in real space and actually gives rise to an attraction). One result is an increase in the phase volume ($\sim \sin\vartheta d\vartheta$), and a second is a change in the nature of the expansion of the effective interaction $\widetilde{\Gamma}(\vartheta)$ near the singularity: While in the case $\alpha = 0$ we have $\widetilde{\Gamma}(\vartheta)$

$\sim x^2 \ln x (x = \pi - \vartheta)$, at nonzero values of α we have $\tilde{\Gamma}(\vartheta) \sim x \ln x (x = \vartheta - \vartheta_c)$. Correspondingly, for the partial components $\tilde{\Gamma}_l$ with large l we have $\tilde{\Gamma}_l \sim I/l^4$ at $\alpha = 0$ and $\tilde{\Gamma}_l \sim I/l^{5/2}$ at nonzero α.

2. As in the absence of a field, an instability of the normal state of a polarized gas is manifested by the appearance of a pole in one of the partial components Γ_l of the complete vertex Γ(Ref. 3). Since s pairing is not possible in the case of a repulsive interaction, we will examine the possibility of P pairing, as in Ref. 2. (A calculation confirms that the transition temperature is highest at $l = 1$.) In a magnetic field there are naturally two transition temperatures, $T_c^{\uparrow\uparrow}(S_z = 1)$ and $T_c^{\downarrow\downarrow}(S_z = -1)$; particles whose spins are directed along the field are the first to pair.

The corresponding calculation is carried out as in Ref. 2, with the one distinction that in the calculation of, for example, $T_c^{\uparrow\uparrow}$, it is necessary to insert the Green's functions of particles with spins directed along the field $G^{\uparrow}$, into the Cooper channel, while $G^{\downarrow}$ must be substituted into the zero-sound channel (which determines the effective interaction $\tilde{\Gamma}$). As a result, for the temperature corresponding to the first instability we find

$$T_c^{\uparrow\uparrow} = T_{c1} \exp\{f(\alpha)/(ap_F)^2\}, \tag{1}$$

where T_{c1} was found in Ref. 2:

$$T_{c1} \sim \epsilon_F \exp\{-5\pi^2/8(2\ln 2 - 1)(ap_F)^2\}, \tag{2}$$

and

$$f(\alpha) = \frac{5\pi^2}{8(2\ln 2 - 1)}(1 - [\delta^3(\frac{1+\delta^3}{2})^{2/3}(1 + \frac{\delta - 1}{3(2\ln 2 - 1)}\Psi_\delta)^{-1}]), \tag{3}$$

where

$$\Psi_\delta = (\delta + 1)[10\ln(\delta + 1) - \delta^2 - 3] + \frac{\delta - 1}{2}(\delta^3 + 2\delta^2 + 8\delta + 4)\ln\frac{\delta + 1}{\delta - 1}$$
$$+ \frac{6}{\delta - 1}\ln\frac{\delta + 1}{2}, \tag{4}$$

and the quantity $\delta = p_F^{\uparrow}/p_F^{\downarrow}$, which is the ratio of the Fermi momenta of the components with spin up and spin down, is related to the polarization by $\delta = (1 + \alpha/1 - \alpha)^{1/3}$. The function $f(\alpha)$ has the following behavior in the specified limiting cases:

$$f(\alpha) = \begin{cases} \dfrac{5\pi^2(7 - 4\ln 2)\alpha}{36(2\ln 2 - 1)^2}, & \alpha \ll 1 \\ -9\pi^2/(8(2)^{2/3}(1 - \alpha)), & 1 - \alpha \ll 1. \end{cases} \tag{5}$$

The decrease in T_c as $\alpha \Rightarrow 1$, which follows from (5), is related in an obvious way to the vanishing of the density of states of particles with spin projection opposite the field in this limit. The competition between this effect and the amplification of the Kohn singularity mentioned above, strictly speaking, leads to the maximum on the $T_c(\alpha)$ curve. A numerical calculation shows that the maximum occurs at $\alpha = 0.48$. The value of $f(\alpha)$ at the maximum is 7.43.

Actually, the absence of an effect in the case $\alpha = 1 (T_c^{\uparrow\uparrow} = 0)$ does not mean that P pairing is impossible, since the vacuum interaction generally has a nonzero P harmonic. The corresponding temperature in a low-density gas, however, contains an extra power of the gas parameter in the exponent[4]: $\ln \varepsilon_F / T_c^{\uparrow\uparrow} \sim (ap_F)^{-3}$. The equations derived here also apply to a low-density gas with an attraction in the case $\mu H \gg T_{c0}$ (the s-pairing temperature at $H = 0$), since the pole in the s channel disappears in such fields.

3. As in Ref. 2, we can extrapolate these equations to ^3He, in which case we have $ap_F = 2$. In this case, $T_c^{\uparrow\uparrow}(\alpha_{\max})$ is greater than T_{c1} by a factor of 6.41. This estimate may be of some interest in connection with the hope to achieve high degrees of polarization in liquid ^3He through a rapid melting of a paramagnetic ^3He crystal (the Castaing-Nozieres method[5]). Approximately the same value of the maximum increase in T_c (a factor of 5) was found in Ref. 6, where the values of the effective interactions in the s and P channels were taken from experiments. Another curious point is that in weak fields the field dependence of the difference between critical temperatures calculated from (1) and from the corresponding expression for $T_c^{\downarrow\downarrow}$,

$$T_c^{\uparrow\uparrow} - T_c^{\downarrow\downarrow} \approx 3.4 \cdot 10^{-9} H(\mathrm{K/G})$$

also agrees fairly well with the experimental value of the width of the region in which the A_1 phase exists.[7]

4. We conclude with a look at a manifestation of the Kohn-Luttinger effect in the 2D case. A distinctive feature of the 2D case is that in the absence of a field the Fourier component of the effective interaction for particles on the Fermi surface (with a δ-function interaction potential) does not have harmonics with $m \neq 0$ (m is the magnetic quantum number). In other words, an effective attraction, because of the Fermi background, does not arise.[8] Incorporating a field changes the situation. We find an interval of φ (the angle between the incoming and outgoing momenta) in which the effective interaction turns out to be a function of φ:

$$\tilde{\Gamma}(\varphi) = \frac{16\pi}{m} f_0^2 \{1 - \mathrm{Re}[1 - \frac{2}{\delta^2(1+\cos\varphi)}]^{1/2}, \tag{6}$$

where in this case we have $\delta = ((1+\alpha)/(1-\alpha))^{1/2}$, and f_0 is the (dimensionless) zeroth harmonic of the scattering amplitude. As a result, with $m \neq 0$ there is an effective attraction, which turns out to have a maximum absolute value in the $m = 1$ case. The expression for the corresponding temperature is

$$\ln \frac{\epsilon_F}{T_c^{\uparrow\uparrow}} = \frac{\delta^2}{(\delta - 1) 8 f_0^2} \tag{7}$$

The temperature T_c reaches a maximum at $\alpha = 0.6$.

In analyzing the result we note that if the Born approximation is valid, i.e., if

$m|U_0| \ll 1$ (U_0 is the zeroth Fourier component of the vacuum potential), then we have $f_0 = -mU_0/4\pi$, and expression (7) is valid for $1 \gg f_0 \gg (p_F r_0)^2$, where r_0 is the range of the vacuum potential. In the opposite limit, $mU_0 \gg 1$, which is typical of the characteristic potentials in the 2D case, f_0 is, with logarithmic accuracy, a universal function of the gas parameter[9] $p_F r_0$: $f_0 = (2\ln p_F r_0)^{-1}$. In this case, the satisfaction of the condition $p_F r_0 \ll 1$ is sufficient to make (7) valid. The value of $T_c^{\uparrow\uparrow}$ at the maximum is

$$T_c^{\uparrow\uparrow} \sim \epsilon_F \exp\{-1/2f_0^2\} = \epsilon_F \exp\{-2\ln^2 p_F r_0\}.$$

The numerical coefficient of $1/f_0^2$ is far smaller than in the 3D case.

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