Ultraviolet behavior in five-dimensional Yang-Mills theory

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The five-dimensional Yang-Mills theory is shown to have an ultraviolet cutoff when a special diagram technique is used, and that it has a fixed ultraviolet point.

Various aspects of a three-dimensional quantum field theory have recently been the subject of active research.[1] This interest stems, in particular, from the fact that the properties of space-time renormalizability of an odd number of dimensions and an even number of dimensions are fundamentally different. These differences manifest themselves in that well-known and currently folkloric fact that in the odd number of dimensions the single-loop integrals are ultraviolet-finite in the sense of analytic continuation. Specifically, in an odd number of dimensions one can subtract ultraviolet divergences at zero momenta even in massless theories. In the even number of dimensions, on the other hand, a subtraction cannot be carried out at zero momenta for the massless theories because of the presence of infrared divergences.

In the present letter we will consider the ultraviolet behavior in the five-dimensional Yang-Mills theory. We will show that a five-dimensional Yang-Mills theory provides an ultraviolet cutoff when a special kind of diagram technique is used and it has an ultraviolet fixed point.

In a $n = 4 + 2\epsilon$ space-time the formula for the β-function is

$$\beta(\alpha, n) = \alpha\epsilon + \beta_4(\alpha),$$

where $\beta_4(\alpha) = -\beta_0\alpha^2 + \beta_1\alpha^3 + ...$is a four-dimensional β-function. Because of asymptotic freedom, the β-function is negative and the equation

$$\beta(\alpha, n) = 0$$

has a nontrivial solution and an ultraviolet fixed point. In the background-field method[3] the simple Ward identities are satisfied and the anomalous dimension of the gauge-field propagator is related to the 4D β-function by the relation

$$\beta(\alpha) = \alpha\gamma(\alpha) . \tag{1}$$

From relation (1) and the renormalization-group equation we find the asymptotic expression for the gluon propagator (here and elsewhere in the text, we use the background-field method)

$$D(p^2) \sim (1/p^2)(p^2/\mu^2)^{-\epsilon} . \tag{2}$$

In the one-loop approximation the odd-dimensional field theory has, as was mentioned above, an ultraviolet cutoff. In a one-loop approximation the solution of the renormalization-group equation

$$p^2 \frac{d\bar{\alpha}}{dp^2} = -\beta_0 \bar{\alpha}^2 + \frac{1}{2}\bar{\alpha} \tag{3}$$

is

$$\bar{\alpha}\left(\frac{p^2}{\mu^2}, \alpha\right) = \frac{1}{2\beta_0 + \left(\frac{p^2}{\mu^2}\right)^{1/2}\left(\frac{1}{\alpha} - 2\beta_0\right)} \, . \tag{4}$$

The corresponding solution of the renormalization-group equation for the gluon propagator is

$$D_{\mu\nu} = \frac{\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)}{p^2} \; \frac{1}{1 + \left(\frac{p^2}{\mu^2}\right)^{1/2}\beta_0 - 2\beta_0} \tag{5}$$

In terms of the Feynman diagrams propagator (5) corresponds to a summation of diagrams of the type in Fig. 1. It is easy to see that when an effective propagator (5) is used, perturbation theory is, according to the counting rules for ultraviolet divergences, a renormalizable theory. It can be shown, moreover, that when an effective propagator (5) is used, there is no ultraviolet divergence to the gluon propagator (more precisely, the ultraviolet divergence can be subtracted at a zero momentum). Because of the simple Ward identities, which are satisfied in the background-field method, this means that the vertex functions also have no divergences, i.e., the β-function for the effective propagator (5) vanishes identically. This can be proved by the mathematical induction method. We must first show, however, that all one- loop diagrams are finite diagrams when propagator (5) is used. The two-loop diagrams in this case do not contain any subdivergences, and they can be provided with an ultraviolet cutoff by subtracting the divergences at zero momenta. The ultraviolet cutoff of n-loop diagrams is proved in a corresponding way.

Note that a rapid decay of propagator (5) is consistent with the postulate that the norm of the state is positive, since the gluon propagator depends on the gauge, and since the longitudinal gluons with an indefinite metric contribute to this propagator (in the background-field method). A study of the spectrum of the model must consider the correlations of the gauge-invariant currents. Thus, for example, in a leading

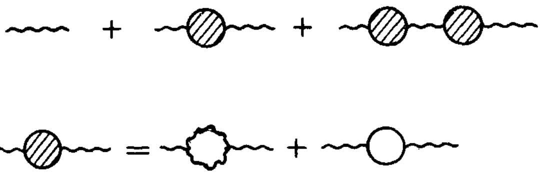

FIG. 1.

"logarithmic" approximation we have

$$D(p^2) = i\int e^{ipx} < 0 \mid T((G^{Q}_{\gamma\nu}(x))^2 \, (G^{Q}_{\mu\nu}(0))^2 \, ,$$

$$\mid 0 > d^4x = kp^2 \sqrt{p^2} \left[\frac{1}{1 + 2\beta_0 [(\frac{p^2}{\mu^2})^{1/2} - 1]} \right]^2 . \qquad (6)$$

At a fixed point

$$D(p^2) \sim \sqrt{p^2}$$

and does not contain states with an indefinite metric.

We have established that a five-dimensional Yang-Mills theory provides an ultraviolet cutoff when effective propagator (5) is used and that the ultraviolet asymptotic behavior of the gluon propagator is described by formula (5).

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