Properties of "dirty" S–S^*–N and S–S^*–S structures with potential barriers at the metal boundaries

A. V. Zaĭtsev
Institute of Radio Engineering and Electronics, Academy of Sciences of the USSR

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The dependence of the current I and conductivity on the voltage V, temperature, and transmission level of the barriers in "dirty" S–S^*–$N(S)$ structures is studied. The presence of two barriers may lead, in particular, to a nonmonotonic, sign-changing dependence of $I - V/R_n$ on the temperature at large values of V. Such a behavior was observed experimentally in high-T_c superconducting point contacts.

In the present letter we report the results of an experimental study of the current in structures of the type S–S^*–$N(S)$ (S^* is a superconductor, the critical temperature $T_c^* < T_c$, and N is the normal metal), in which the boundaries between the metals have low-transmission potential barriers. The solution of this problem is of current interest because such structures, which are based on high-T_c superconductors and ordinary metals, are now being studied experimentally. The boundaries between these structures generally have a low transmission level (this problem is discussed in Ref. 1, in which other experimental studies are cited). The current cannot be calculated by means of a expansion in powers of the transmission, regardless of its low level. A microscopic method must therefore be used since the tunnel–Hamiltonian method is inapplicable. A similar problem, we might note, was solved in Ref. 2 in the case of Josephson S–S^*–S junctions at equilibrium (at $V = 0$).

In developing the theory we studied as a model of the S–S^*–N and S–S^*–S structures a bridge of length d, whose boundaries between the edge and the constriction are characterized by the transmission coefficients D_1 and D_2. Calculations were carried out for "dirty" metals, where the mean free path (l^* in S^*) is the shortest of all the characteristic lengths in each metal. The system of semiclassical equations[3-5] and its boundary conditions[5] in this case can be obtained for an angle-integrated matrix Green's function $\check{G}$ which is comprised of a retarded Green's function $\hat{G}^R$, advanced Green's function $\hat{G}^A$, and the Green–Keldysh function $\hat{G} = \hat{G}^R \hat{f} - \hat{f} \hat{G}^A$, where $\hat{f} = f_l + f_t \hat{\tau}_z, f_l, f_t$ is the distribution function.[4] In the dirty limit, the boundary conditions[5] of the equation for G can be reduced to the form[2]

$$4l^*(\check{G} d\check{G}/dx)(x_j) = 3\mathcal{D}_j(-1)^j[\check{G}(x_j), \check{G}_j], \quad \mathcal{D}_j = \langle \kappa_j D_j/(1-D_j) \rangle = (l^*/d)(R_*/R_j), \tag{1}$$

where $x_1 = +0, x_2 = d - 0$, and $\check{G}_{1(2)}$ are the equilibrium Green's functions for the edges, $R_* = d/\sigma_* A$ is the resistance of the S^* metal in the normal state, which is governed by the conductivity σ_*, A is the cross-sectional area, R_j is the resistance of the jth barrier (at $T > T_c$), and $\kappa_j = p_c^k/p_F^k$; here the superscript $k(= *$ or $j)$ corresponds to a metal with the smaller value of the Fermi momentum, p_F^k.

The current in the S–S^*–N structures will be calculated under the assumption that $(\hbar = 1)$

$$d \ll (l^* v_F^*/T_c)^{1/2}, \quad v_F^* \tau_{in} /D_j; \quad R_j \gg R_*, \tag{2}$$

where τ_{in} is the time scale of the inelastic relaxation in S^*. If condition (2) is satisfied, all the functions in the region $0<x<d$ in the first approximation do not depend on x. Assuming that the potential and the phase of the order parameter in S vanish, we find the following expressions from the equations for $\hat{G}^R$ (Ref. 4) and Eq. (1) (in the steady-state case)

$$\begin{gathered}\hat{G}^R = g^R \hat{\tau}_z + \hat{f}^R = [u^R \hat{\tau}_z + i\ (\delta^R \hat{\tau}_y + \ \Delta_* \sin\varphi\, \hat{\tau}_x)]/\zeta^R, \ u^R = \epsilon + i(\epsilon_1 g_s^R + \epsilon_2),\\ \zeta^R = [(u^R)^2 - (\delta^R)^2 - (\Delta_* \sin\varphi)^2]^{1/2}, \ \delta^R = \Delta_* \cos\varphi + i\epsilon_1 f_s^R, \ \epsilon_j = D_j v_F^*/4d,\end{gathered} \tag{3}$$

where g_s^R and f_s^R are the Green's functions of the superconductor S, Δ_* is the modulus, and φ is the order-parameter phase of S^*, which in our reference frame coincides with the phase difference at the S–S^* boundary. Taking into account the relationship between the order parameter and the function $\hat{f}^R \hat{f} - \hat{f}\hat{f}^A$ (Refs. 3 and 4), we find the equations for Δ_* and φ which in the weak-coupling model are given by

$$\lambda\Delta_* = \epsilon_1(\alpha\cos\varphi - \beta\sin\varphi), \ \tan\varphi = (\alpha\rho - \lambda\beta)/(\lambda\alpha + \beta\rho), \tag{4}$$

where $\lambda = \ln(T/T_{co}^*) - \int_0^\infty d\epsilon[f_l \operatorname{Re}(1/\zeta^R) - n(\epsilon)/\epsilon]$, $\rho = -\int_0^\infty d\epsilon \operatorname{Im}(1/\zeta^R) f_t$, $\alpha = -\int_0^\infty d\epsilon \operatorname{Im}(f_s^R/\zeta^R) f_l$, $\beta = \int_0^\infty d\epsilon f_t \operatorname{Re}(f_s^R/\zeta^R)$, $n(\epsilon) = \operatorname{th}(\epsilon/2T)$, and T_{co}^* is the critical temperature in the absence of decoupling. It can be shown that this condition is satisfied in the range of the parameters T and ϵ_j in which the function λ is greater than zero. In particular, at $(T, \epsilon_j) > T_{co}^*$ it remains a steady-state solution for any (steady-state) currents. This assertion also holds for the superconducting current $I_S (<I)$ which flows across the S–S^* boundary. This current is given by the expression $I_s = \Delta_* \alpha \sin\varphi /eR_1$. Here we restrict the discussion to the steady-state case indicated above. Solving the system of equations for f_l and f_t (Ref. 4) and using (2), we obtain the following expressions for the current and the distribution functions:

$$I = (1/eR_n) \int_0^\infty d\epsilon \{F_1(\epsilon) n_-(\epsilon) + F_2(\epsilon)[n_+(\epsilon) - n(\epsilon)]\}, \tag{5}$$

where $R_n = R_1 + R_2$ is the resistance of the structure in the normal state, $n_\pm(\epsilon) = [n(\epsilon + eV) \pm n(\epsilon - eV)]/2$,

$$\begin{gathered}F_1 = [(1+a)\nu - M_t F_2/\eta], \ F_2 = a(1+a)\eta\nu^2/[M_t M_l + \eta^2], \ \eta = \Delta_* \sin\varphi \operatorname{Re} f_s^R \operatorname{Im} 1/\zeta^R,\\ M_l = \nu(\nu_s + a) - \operatorname{Re} f_s^R \operatorname{Re}\delta^R/\zeta^R, \ M_t = \nu(\nu_s + a) - \operatorname{Im}[\delta^R \operatorname{Re}\delta^R + (\Delta_* \sin\varphi)^2]/\epsilon_1\zeta^R,\end{gathered} \tag{6}$$

$$f_t = n_- - [F_1 n_- + F_2(n_+ - n)]/\nu(1+a), \ f_l = n + [\eta f_t + a\nu(n_+ - n)]/M_l. \tag{7}$$

Here $\nu_{(s)} = \operatorname{Re} g_{(s)}^R$, and $a = D_2/D_1 = R_1/R_2$. Substituting (7) into (4), we can in principle find the functional dependence $\Delta_*(V)$ and $\varphi(V) = -\varphi(-V)$, and calculate the current from (5). Analytic expressions can be obtained in several limiting

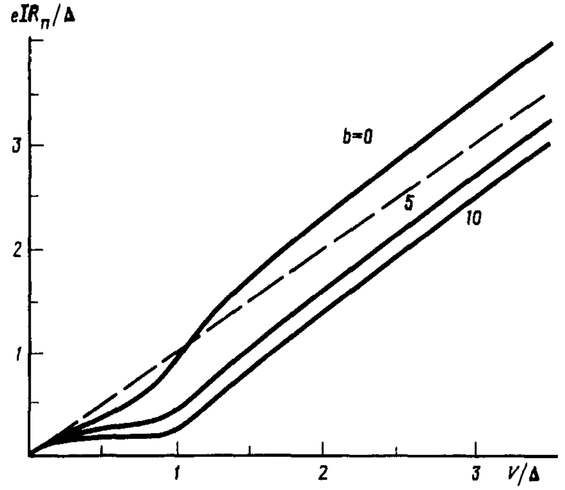

FIG. 1. The current–voltage characteristics of the S–N–N' structure at $T=0$ and $R_1=R_2$ for various $b=4d\Delta/Dv_{F*}$. The dashed curve corresponds to the dependence $I=V/R_n$.

cases. The simplest result is found from (5) and (6) for the S–N–N' structure, where N is a metal with $T_{co}^N=0$ $(\Delta_N=0)$. In this case $F_2=0$ and

$$F_1=(1+a)/\{1/\nu+a/[\nu_s\nu+\epsilon_1 \mathrm{Im} f_s^R \mathrm{Re}(f_s^R/\zeta^R)]\}, \quad (8)$$

where ζ^R is given by expression (3) for $\Delta_*=0$. Let us analyze the results that follow from (5) and (8) for the case in which the Green's functions of a superconductor S are given by the standard expressions of the BCS theory: $g_s^R=\epsilon f_s^R/\Delta=\epsilon/[(\epsilon+i0)^2-\Delta^2]^{1/2}$. Figure 1 shows the I–V curves which were calculated numerically for this case at $a=1$, $T=0$, and various values of the parameter $b=\Delta/\epsilon_{1(2)}$. At $eV\gg\Delta$ the I–V characteristic (for arbitrary T) has the usual form[5,6]

$$I=V/R_n+I_0\tanh(eV/2T),\quad eR_nI_0=\int_0^\infty d\epsilon[F_1(\epsilon)-1]\equiv\Omega(b_1,a),\quad (b_j=\Delta/\epsilon_j). \quad (9)$$

In contrast with the point contacts with a "direct" conductivity[5,6] $(R_j=0)$ and with a single barrier,[5] however, the current I_0 may be either positive or negative[1)] and may have a temperature dependence which differs markedly from $\Delta(T)$. These results also apply to S–N–S structure in which $eI_0R_n=\Omega(b_1,a+\Omega(b_2,1/a)$. A plot of $I_0(T)$ for the case $a=1$ is shown in Fig. 2. We see that with a decrease in the transition D [an increase in $b_0=\Delta(0)/\epsilon_{1(2)}$,] the function $I_0(T)$ becomes nonmonotonic oscillating function. Such plots of $bI_0(T)$ were observed[2)] in experiments with Josephson junctions with microcracks[8] (whose structure seems to correspond to the S–N–S model) which were based on various high-T_c superconductors. A plot of $v_0\equiv eI_0R_n/2\Delta$ vs $b=(b_1+b_2)/2$ for an S–N–S structure for two values of a is shown in the inset in Fig. 2.

Worth noting are the plots $\sigma(V)=dI/dV$ (at low temperatures) and

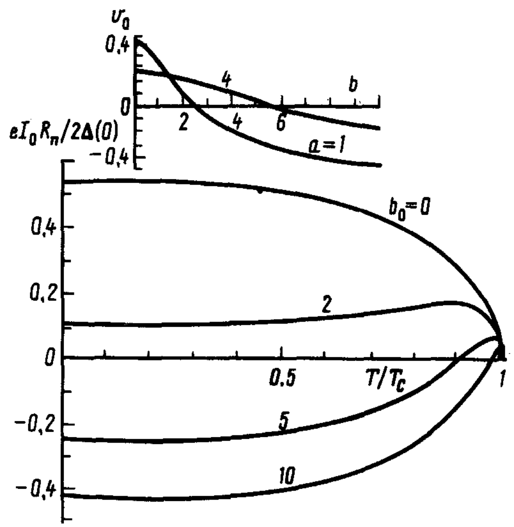

FIG. 2. Temperature dependence of I_0 in the case $R_1 = R_2$ for various $b_0 = 4d\Delta(0)/Dv_F^*$. The inset shows the dependence $eR_nI_0/2\Delta \equiv v_0 \quad (b;a) = v_0 \quad (b;\ 1/a)$ for an S–N–S structure, where $b = 4d\Delta\ (1/D_1 + 1/D_2)/v_F^*$; at $a = 1[v_0(0;1) \approx 0.53;\ v_0(\infty;1) = -2/3]v_0$ determines the value of eR_nI_0/Δ in S–N–N' structures.

$\sigma_0(T) = \sigma(V = 0, T)$ for an S–N–N' structure (shown in Fig. 3), whose qualitative form differs appreciably from those found in Refs. 5 and 6, and also from the known plots which are characteristic of S–N tunnel junctions (in which the conductivity at zero has a minimum, rather than a peak).

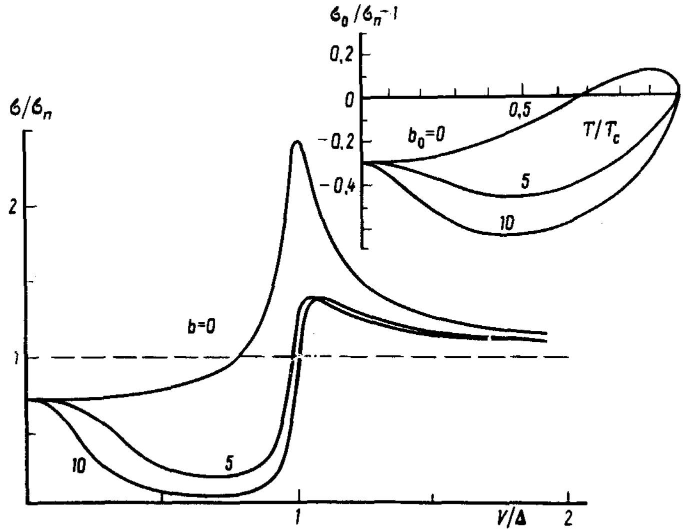

FIG. 3. The dependence $\sigma(V)$ in S–N–N' structures at a low temperature $T = \Delta/10$ for $R_1 = R_2$. The inset shows the dependence $\sigma_0(T) = \sigma(V = 0, T)$ for various values of b_0.

For the S–S^*–N structures the analytic expressions can be derived from (4)–(7) in several limiting cases. In particular, worth noting is the region of T, $eV \ll \Delta$, for which a simple formula can be obtained when the decoupling parameter in $S^*/\gamma_* = \epsilon_1 + \epsilon_2 + 1/\tau_s^* \gg \epsilon_j$, Δ_* [$\tau_s^{(*)}$ is the time required for the spin to flip; this time is determined by the scattering by magnetic impurities in $S^{(*)}$, $\gamma = 1/\tau_s \ll \Delta$]

$$I = \mathrm{Im}\Psi(\Gamma_* + iV/2\pi T)[1 + 2(\Delta_*/\epsilon_1)\cos\varphi + (\Delta_*/\epsilon_1)^2](\epsilon_1 + \epsilon_2)/aeR_n, \tag{10}$$

where $\Gamma_* = 1/2 + \gamma_*/2\pi T$, $\Delta_*(V)$, and $\cos\varphi(V)$ are determined from (4), with allowance for the fact that

$$\lambda \approx \ln T/T_{co}^* + \mathrm{Re}\Psi(\Gamma_* + iV/2\pi T) - \Psi(1/2), \quad \rho \approx -\beta \approx \mathrm{Im}\,\Psi(\Gamma_* + iV/2\pi T),$$
$$\alpha \approx \int_0^\Delta d\epsilon n_+(\epsilon)\,\Delta\epsilon/(\epsilon^2 + \gamma_*^2)(\Delta^2 - \epsilon^2)^{1/2}, \tag{11}$$

where $\Psi(z)$ is the digamma function. It follows from (10) and (11) that at γ_*, $T < eV \ll \Delta$ there is a region in which $\sigma(V) < 0$ and the I–V characteristic has an N-shaped form, i.e., voltage surges and a hysteresis are seen in the specified current mode. This conclusion holds even when $\tau_s^* = \infty$.

In the region of strong voltages $eV \gg \Delta$, T, and $\gamma_{(*)}$ for the S–S^*–N structures we find

$$I - V/R_n = [I_0 + I_*\Phi(\ln(e|V|/\epsilon_*))]\,\mathrm{sign}\,V \equiv \delta I(V;\epsilon_1, a), \tag{12}$$

where I_* may be either higher than or lower than the current I_0 given by expressions (8) and (9); at $x \gg 1$ $\Phi(x) = 1/x$, ϵ_* is a characteristic energy which depends on Δ, T_{co}^*, T, and $\gamma_{(*)}$. In particular, if the decoupling in S is strong ($\gamma \gg \Delta$), we have

$$eR_nI_0 = [\Delta^2\pi a/8\gamma(1+a)][2\epsilon_1/(\gamma+\gamma_*) - (\Delta/2\gamma)^2/(1+a)],$$
$$eR_nI_* = \pi\Delta^2\epsilon_1[\Psi(\Gamma) - \Psi(\Gamma_*)]/(1+a)(\gamma^2 - \gamma_*^2), \quad \epsilon_* = 0.88T_{co}^*[y(\gamma_*/2\pi T)T_{co}^*/T]^{1/a}, \tag{13}$$

where $y(x) = \exp[\Psi(1/2) - \Psi(1/2 + x)]$, $y(x) \approx 1/x$ for $x \gg 1$. In S–S^*–S structures at high voltages $I - V/R_n = \delta I(V;\epsilon_1, a) + \delta I(V;\epsilon_2, 1/a)$. It follows from (13) that in contrast with the case of weak decoupling, a strong decoupling gives rise to a transition from the dependence $I_0 \sim \Delta^2$ to the dependence $I_0 \sim -\Delta^4$ as the barrier transmission decreases.

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[1)] In the analysis of another current-transport mechanism in Ref. 7a similar conclusion was drawn in a study of sandwiches with a semiconducting layer.
[2] I am grateful to B. A. Aminov for furnishing the experimental results before their publication.

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