Dielectric correlations and δ-type superconductivity in a system with Coulomb interaction

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A general expression has been derived for the effective potential of the Coulomb interaction between electrons in the superconducting channel, with allowance for the dielectric correlations in the weak-interaction limit. The conditions under which an $s + id$-type superconductivity can occur have been determined. A new state, which competes with the RVB phase in the strong-interaction limit, is discussed.

Experimental data showing that there are strong magnetic correlations in high-T_c superconductors and a greatly weakened isotope effect have stimulated theoretical study of superconducting pairing in models with a Coulomb interaction between electrons.[1] The possibility of a d-type superconductivity has been demonstrated in the strong-interaction limit[2,3] and in the weak initial (seed) interaction limit.[4–6] A strong-interaction model also incorporates s-type superconducting correlations.[2] The structure of the ground state of the superconducting phase in the strong-interaction model is now a subject of lively discussion.[3,7] The determination of this structure requires the study of the interface of electron–electron and electron–hole correlations.[3,8] In the present letter we solve the problem in the weak-interaction limit and we show that allowance for the complex structure of the dielectric correlations in the weak-interaction limit gives rise to superconducting s pairing due to the initial (seed) repulsion.

1. Let us consider a model with congruent regions of the Fermi surface and a screened Couloumb interaction between electrons. The Hamiltonian of the model is

$$\hat{\mathcal{H}} = \sum_{i,\mathbf{k},\alpha} \epsilon_i(\mathbf{k}) a^+_{i\mathbf{k}\alpha} a_{i\mathbf{k}\alpha} + \sum_{i,j,l,m} g_{ijlm} a^+_{i\mathbf{k}\alpha} a^+_{j\mathbf{k}'\beta} a_{l\mathbf{k}'\mathbf{q}\beta} a_{m\mathbf{k}'+\mathbf{q}\alpha} , \tag{1}$$

where α and β are the spin indices, $i, j, l, m = 1,2$ are the congruent regions of the Fermi surface, and $\epsilon_{1,2}(\mathbf{k}) = \mu \pm \epsilon(\mathbf{k})$; here μ is the parameter of the incongruence due, for example, to doping or anisotropy. In the case of a square Fermi surface, which is widely discussed in the study of high-T_c superconductivity, the congruent regions of the Fermi surface 1 and 2 are separated by one-half the reciprocal lattice vector $\mathbf{Q}$: $\epsilon_1(\mathbf{k}) = \epsilon_2(\mathbf{k}+\mathbf{Q})$, $\epsilon_i(\mathbf{k}) = \epsilon_i(-\mathbf{k})$, and belong to one zone. In general, indices 1 and 2 may refer to different zones with different symmetries of the wave functions.

The interaction $g_2 \equiv g_{1122}$ in (1) (these are umklapp processes in a single-band model) confuses the Cooper channel with the zero-sound channel, in which the logarithmic singularity stems from the "nesting." However, because of integration over the angles, there is an effective decoupling of the "parquet" and the ground-state structure qualitatively corresponds to the results of a mean-field approximation.[4]

Let us consider the dielectric correlations by means of a canonical transformation which eliminates the singularity in the zero-sound channel ($\alpha = ua_1 + va_2$). We assume that the chemical potential μ in the reconstructed phase is situated under the dielectric gap, and that there exist charge carriers (holes) at $T=0$. For the superconducting order parameter, $\tilde{\Delta} \sim \langle \alpha\alpha \rangle$ we find the following equation:

$$\tilde{\Delta}(\mathbf{k}) = -\int V(\mathbf{k},\mathbf{k}') \frac{\tilde{\Delta}(\mathbf{k}')}{\omega(\mathbf{k}')} \operatorname{th} \frac{\omega(\mathbf{k}')}{2T} \frac{d\mathbf{k}'}{(2\pi)^3}, \tag{2}$$

where $\omega(\mathbf{k}) = \sqrt{(\mu - E(\mathbf{k}))^2 + |\tilde{\Delta}(\mathbf{k})|^2}$, $E(\mathbf{k}) = \sqrt{\epsilon^2(\mathbf{k}) + |\hat{\Sigma}(\mathbf{k})|^2}$, and $\hat{\Sigma}(\mathbf{k})$ is the dielectric order parameter. In general $\hat{\Sigma}(\mathbf{k})$ contains singlet (Σ) and triplet (Σ^+) real (Σ^R) and imaginary (Σ_I) components:[9] $\hat{\Sigma} = \Sigma_R + i\Sigma_I + \sigma(\Sigma_R^t + i\Sigma_I^t)$. The effective interaction $V(\mathbf{k},\mathbf{k}')$ is given by

$$\begin{gathered} V(\mathbf{k},\mathbf{k}') = \Gamma_0(u^2u'^2 + v^2v'^{*2}) + \sigma\Gamma_2(v^2u'^2 + u^2v'^{*2}) + 2(\Gamma_1 + \sigma\tilde{\Gamma}_2)uvu'v'^*, \\ u', v' \equiv u(\mathbf{k}'), v(\mathbf{k}'); \quad \Gamma_i \equiv \Gamma_i(\mathbf{k},\mathbf{k}',\mathbf{k}+\mathbf{k}'), \end{gathered} \tag{3}$$

where $\Gamma_i(\mathbf{k},\mathbf{k}',q)$ are the complete vertices (at zero frequency) (Fig. 1) which correspond to the bare charges $g_0 \equiv g_{iiii}, g_1 = g_{1221}$, $g_2 = g_{1122}$, and $\tilde{g}_2 = g_{1212}$ in (1). The vertices Γ_1, Γ_2, and $\tilde{\Gamma}_2$ in the zero-sound channel have a pole at the point where the long-range dielectric order is established ($T = T_D$) and a maximum near the temperature T_D which characterizes the formation of the short-range dielectric order. In the last case relation (3) can be used to describe the electron–electron interaction if the correlation length of the superconducting order parameter ξ_0 is shorter than the size of the short-range order regions. In deriving (3) we assumed that $T \ll T_D$, T_D^*, and u and v in (3) are the canonical transformation coefficients

$$u^2, |v|^2 = \frac{1}{2}\left(1 \mp \frac{\epsilon(\mathbf{k})}{E(\mathbf{k})}\right), \quad uv = \frac{1}{2} \frac{\hat{\Sigma}(\mathbf{k})}{E(\mathbf{k})}. \tag{4}$$

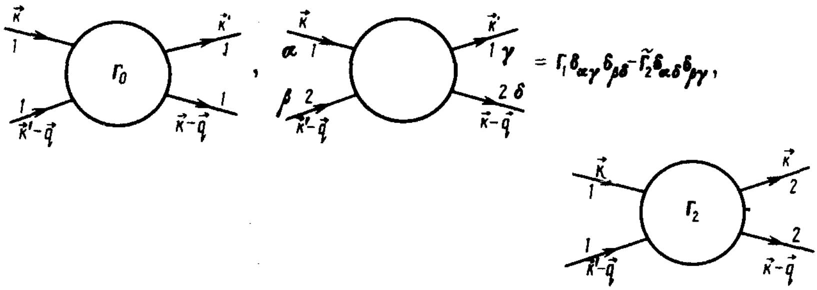

FIG. 1.

The factor σ in (3) assumes the value of -1 for the triplet dielectric order parameter and 1 for the singlet order parameter. The sign of the effective interaction in (3) is determined by the spin and phase structure of the dielectric order parameter and by the ratio of the amplitudes of the various Coulomb scattering processes.

2. As an example, let us consider a single-band model with planar regions of the Fermi surface. The results for a two-band model will be considered in a separate paper. In the simple case of a single-center Hubbard interaction all the bare charges in (1) are $g_0 = g_1 = g_2 = \tilde{g}_2 = U$. In the random-phase approximation for the vertices we have

$$\Gamma_0 = V^- + V^+, \quad \Gamma_2 = V_Q^- + V_Q^+, \quad \Gamma_1 = V^- + V_Q^+, \quad \tilde{\Gamma}_2 = V_Q^- + V^+, \tag{5}$$

where

$$V^-(\mathbf{q}) = \frac{U}{1 - U^2\chi^2(\mathbf{q})}, \quad V^+(\mathbf{q}) = \frac{U^2\chi(\mathbf{q})}{1 - U\chi(\mathbf{q})},$$

$$V^\pm(\mathbf{q}) = V^\pm(\mathbf{k} \pm \mathbf{k}'), \quad V_Q^\pm = V^\pm(\mathbf{k} \pm \mathbf{k}' + \mathbf{Q}),$$

here $\chi(\mathbf{q})$ is the zero-sound loop. We obtain from (3) and (5) the expression for the effective interaction

$$V = V^-(u'u + v'^*v)^2 + \sigma V_Q^-(u'v + v'^*u)^2 + V^+(u'u + \sigma v'^*v)^2 + \sigma V_Q^+(u'v + \sigma v'^*u)^2. \tag{6}$$

Physical ordering is characterized by the structure of the dielectric order parameter. It is important below that the current states: the orbital antiferromagnetism [the singlet imaginary order parameter $\Sigma = i\Sigma_I(\mathbf{k})$] and the state with the spin current [the triplet imaginary parameter $\Sigma = i\sigma\Sigma_I^t(\mathbf{k})$], should be described by d-type parameters (see e.g., Ref. 10): $\Sigma_I, \Sigma_I^t = \Sigma_d(\mathbf{k}) = \Sigma_d(\cos k_x - \cos k_y)$. For a single-component dielectric order parameter Eq. (2) with potential (6) has a d-type superconducting solution: $\tilde{\Delta}_d(\mathbf{k}) = -\tilde{\Delta}_d(\mathbf{k} + Q)$, which was considered in the case of a spin-density wave ($\Sigma = \sigma\Sigma_R^t = \text{const}$) in Refs. 5 and 6. Against the background of a charge-density wave (CDW: $\Sigma = \Sigma_R = \text{const}$) or the spin-current state ($\Sigma = i\sigma\Sigma_I^t$) the superconducting order parameter satisfies even the relation

$\widetilde{\Delta}(\epsilon(\mathbf{k})) = -\widetilde{\Delta}(-\epsilon(\mathbf{k}))$, and the effective attraction potential

$$V_{\text{CDW}} \approx \frac{\mu^2 - \Sigma^2}{\mu^2}(V^- + V^+ - V_Q^- - V_Q^+) \tag{7}$$

vanishes in the half-occupation limit ($\mu = \Sigma$). If, on the other hand, a spin-density wave (or a charge-density wave) and a spin-current (or orbital antiferromagnetism) coexist in the dielectric phase (which generally requires going beyond the scope of a single-center model[10]), then we encounter a $s + id$-type superconducting parameter

$$\widetilde{\Delta}(\mathbf{k}) = \widetilde{\Delta}_s(\mathbf{k}) + i\widetilde{\Delta}_d(\mathbf{k}), \quad \widetilde{\Delta}(\mathbf{k}+\mathbf{Q}) = \widetilde{\Delta}^*(\mathbf{k}), \tag{8}$$

where we have in the case of the coexistence of a spin-density wave and a spin-current state

$$\begin{aligned} \widetilde{\Delta}_d(\mathbf{k}) &= (1 - 2u^2(\mathbf{k})\sin^2\theta(\mathbf{k}))\Delta_d(\mathbf{k}) + u^2(\mathbf{k})\sin 2\theta(\mathbf{k})\Delta_s, \\ \widetilde{\Delta}_s(\mathbf{k}) &= u^2(\mathbf{k})\sin 2\theta(\mathbf{k})\Delta_d(\mathbf{k}) + (1 - 2u^2(\mathbf{k})\cos^2\theta(\mathbf{k}))\Delta_s, \end{aligned} \tag{9}$$

here $\Sigma(\mathbf{k}) = |\Sigma(\mathbf{k})|e^{i\theta(\mathbf{k})}$, and $\Delta_d(\mathbf{k})$ is a solution of Eq. (2) with the potential

$$V_{\text{SDW}} \approx V^- - V_Q^- \tag{10}$$

The parameter $\Delta_s \approx \int d\mathbf{k}'/(2\pi)^3(V^- + V^+ + V_Q^- + V_Q^+)\Delta_d(\mathbf{k}')\Sigma_R^+\Sigma_d(\mathbf{k}')/\mu^2\omega(\mathbf{k}')\tanh(\omega(\mathbf{k}')/2T) \approx \text{const}$, $(\Sigma_d \ll \Sigma_R')$.

For a physical interpretation of the mechanism which is responsible for the superconductivity, it is important that a d-type solution with an effective potential

$$V_{T > T_D} = \Gamma_0 - \Gamma_2 = V^- + V^+ - V_Q^- - V_Q^+ \tag{11}$$

exist even above the temperature at which the dielectric order appears when the order parameter is $\Sigma = 0$. In terms of the initial (seed) operators, it is an antisymmetric solution $\Delta_{11} = -\Delta_{22} = \Delta_a$ [$\Delta_{ii} \sim \langle a_i a_i \rangle$, $\Delta_a = \Delta_d B(9)$]. The attraction in (11), as in (6), involves an exchange of electron-hole (exciton) excitations, which are responsible for the establishment of the dielectric order[11] (spin fluctuations at $U > 0$; Refs. 5, 6, and 11). It can be shown that the dielectric order parameter increases upon the appearance of a superconducting condensate $(\partial\Sigma_R'/\partial\Delta^2|_{\Delta=0} > 0)$. There is therefore no reason to interpret the Coulomb superconductivity in terms of "spin bags" or some other "bags,"[6] whose description is based on the dependence of Σ on doping. A change in the symmetry of the wave functions as a result of transition to an insulating state, which is described by the coherence factors, manifests itself in that a spin-density wave is conducive to d-pairing (an antisymmetric solution), while a charge-density wave suppresses it[12] [cf. Eqs. (7) and (10)].

3. In the lattice-site representation the orbital antiferromagnetism state is characterized by a nonzero "flux" Φ, which builds up as it traces out a path along the closed circuit of the square which contains four nearest lattice sites. In the limit of strong

repulsion by a center, at precisely one-half the occupation, we have an $SU(2)$ equivalence of $s+id$ superconducting solution and the phase with a flux $\Phi=\pi$ (Ref. 13). In the weak-interaction limit in the absence of $SU(2)$ invariance we show that an $s+id$ structure of a superconducting condensate (8) is conserved if the current component of the dielectric order is taken into account. This conclusion is valid, however, even in the strong-interaction limit, since it can be presented in the symmetric confirmation form. Regardless of the model used, the free-energy functional has an invariant

$$\delta F \sim (\Delta_s \Delta_d^* - \Delta_s^* \Delta_d)(\Sigma^2 - \Sigma^{*2}),$$

which gives the required structure of the solution.

As follows from the results obtained above, the most favorable conditions for the formation of the $s+id$ phase occur when the spin-density wave and the spin-current state coexist ($U>0$). In the lattice-site representation the spin-current state is described by an alternating triplet, imaginary hop component $\langle c_{i\sigma}^+ c_{j\sigma}\rangle_a \sim \chi_{ij}^\sigma = i|\chi|(-1)^{j_x+j_y} = -\chi_{ij}^{-\sigma}$. In the strong-interaction model[14] a natural generalization of the spin-current state is the parameter

$$\chi_{ij}^\alpha = |\chi| \exp[i(-1)^\alpha(-1)^{j_x+j_y}\theta], \qquad (12)$$

where $\alpha=1,...,n$ is the color index. In the limit $n\to\infty$, the spectrum and the energy of the state with the current with color index (12) and the phase with a flux [and other phases which differ from (12) by their multiplicity] are in agreement. In contrast with the phase with a flux,[7] state (12) is invariant under the time reversal. In the strong-interaction models state (12) has not, to the best of our knowledge, been considered previously, and its study is clearly of interest.

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[1] P. W. Anderson, Science **235**, 1196 (1987).
[2] F. C. Zhang *et al.*, Supercond. Sci. Tech. **1**, 36 (1988).
[3] G. Kotliar and J. Liu, Phys. Rev. B **38**, 5142 (1988).
[4] I. E. Dzyaloshinskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **46**, S110 (1987) [JETP Lett. **46**, S93 (1987); I. E. Dzyaloshinsskiĭ and V. M. Yakovenko, Zh. Eksp. Teor. Fiz. **94**, 344 (1988) [Sov. Phys. JETP **67**, 413 (1988)].
[5] Z. Y. Weng *et al.*, Phys. Rev. B**38**, 6561 (1988).
[6] J. R. Schrieffer *et al.*, Phys. Rev. B**39**, 11633 (1989).
[7] X. G. Wen *et al.*, Phys. Rev. B**39**, 11413 (1989).
[8] Yu. V. Kopaev, Usp. Fiz. Nauk **159**, 567 (1989).
[9] B. I. Halperin and T. M. Rice, Solid State Commun. **21**, 115 (1968).
[10] H. J. Schulz, Phys. Rev. B**39**, 2940 (1989).
[11] M. T. Beal-Monod *et al.*, Phys. Rev. B**34**, 7716 (1986).
[12] A. A. Gorbatsevich *et al.*, Phys. Lett. A**125**, 149 (1987).
[13] I. Affleck *et al.*, Phys. Rev. B**38**, 745 (1988).
[14] J. B. Marston and I. Affleck, Phys. Rev. B**39**, 11538 (1989).

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