Natural flicker noise (the "1/f noise") and superconductivity

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The existence of superconductivity is linked with the existence in the system of charged Bose particles of the current and magnetic field fluctuations with a $1/f$ spectrum. The dissipation and intensity of the current source are proportional to the frequency. Because of this circumstance, the "constant" components of the current and magnetic field decay only in a time on the order of the lifetime of the spectrum.

Natural flicker noise. The $1/f$ noise has been observed in various systems, but so far no single viewpoint concerning its nature has been developed.[1] We will base our discussion on the analysis of the natural flicker noise (which is traceable to the "atomic" structure) presented in Refs. 2 and 3. This analysis was used in Ref. 4 to explain the presence of $1/f$ noise in music, which was observed in Ref. 5. In Refs. 2 and 3 the calculation of $1/f$ noise reduces to the solution of the equation which describes the diffusion process of a certain physical quantity $n\ (R,t)$ at the times $\tau_{obs} \gg \tau > \tau_D$. Here $\tau_D = L^2/D$ is the diffusion time, and L is the length scale of the sample. For the flicker–noise region the Langevin equation for the Fourier component $n(\omega,\mathbf{k})$ is

$$(-i\omega + Dk^2)n(\omega,k) = y(\omega,k); \quad \omega_{min} \ll \omega \ll \tau_D^{-1}, \tag{1}$$

$$(yy)_{\omega,\mathbf{k}} = 2Dk^2 n_{eff} \exp\left[-\frac{L_\omega^2 \mathbf{k}^2}{2}\right]; \qquad L_\omega^2 = D/\omega, \quad n_{eff} = AV_\omega \langle \delta n_V \delta n_V \rangle .$$

The minimum frequency ω_{min} is determined by the observation time τ_{obs} which is much shorter than the lifetime of the system: $\tau_{obs} \ll \tau_{life}$. If the condition $\tau \gg \tau_D$ is satisfied, the "particle" has time to diffuse repeatedly and it covers a volume $V_\omega = L_\omega^3 \gg V(L_\omega = \sqrt{D/\omega})$. Because of this circumstance, the strength of the Langevin source is proportional to the effective particle density. $\langle \delta n_V \delta n_V \rangle$ is a simultaneous correlator of fluctuations which are averages over the volume V of the sample. For an ideal gas it is equal to n/V. The constant A is found from the normalization condition.

In Eq. (1) the dispersion of the Gaussian distribution in $\mathbf{k}$ is proportional to the frequency ω, which allows us to make the substitution $\mathbf{k}^2 \to \bar{\mathbf{k}}^2 = \omega/D$ in this equation and to transform to an equation for the function $n(\omega)$. From this equation we derive an expression for the spectral density of the flicker noise[2,3]

$$(\delta n \delta n)_\omega = \frac{\pi}{\ln(\tau_{obs}/\tau_D)} \frac{\langle \delta n_V \delta n_V \rangle}{\omega}, \quad \tau_{obs}^{-1} \ll \omega \ll \tau_D^{-1}. \tag{2}$$

This expression can be represented in the Hooge form.[1]

Natural flicker noise and superconductivity. Before the discovery of high-T_c superconductivity it was assumed that superconductivity theory, at least in its basic form, is complete. New questions have now been raised, in particular, concerning the physical reason for the disappearance of electrical resistance to the flow of charged Bose particles such as the Cooper pairs. We will show that the existence of superconductivity is related to the existence of the $1/f$ noise in the fluctuations of current and magnetic field. This relationship stems from the fact that in the Langevin equations for the current and magnetic field in the case of a flicker noise the noise dissipation and strength of the noise source are proportional to $|\omega|$. Because of this circumstance, the "constant" components of the current and magnetic field—the Fourier components, are clearly seen at frequencies which are determined by the lifetime of the system: $\omega = \tau_{life}^{-1}$. Consequently, the widths of the spectra of the constant components are on the order of τ_{life}^{-1}. Against the background of the constant components there is a flicker noise whose lower boundary is determined by the observation time. As a rule, we have $\tau_{obs} \ll \tau_{life}$.

We are considering here only the states which are much lower than the phase transition point at which a system of charged Bose particles forms. To calculate the low-frequency fluctuations near the transition, we must introduce the appropriate Langevin sources into the Ginzburg-Landau-Gor'kov equations or into the corresponding equations of the Bardeen-Cooper-Schrieffer theory.

Let us consider two examples of dissipative equations.

1. The dissipation is caused by the viscosity with the viscosity coefficient η. For low currents and low magnetic fields the equation for the curl of the electric current is

$$\frac{\partial}{\partial t}\left(\vec{\Omega} + \frac{e^2 n_s}{mc}\mathbf{B}\right) = D\frac{\partial^2 \vec{\Omega}}{\partial \mathbf{R}^2}, \qquad \vec{\Omega} = \mathrm{curl}\,\mathbf{j}, \qquad D = \eta/\rho. \tag{3}$$

2. The dissipation is characterized by an effective collision frequency ν. The magnetic-field equation in this case can be reduced to the form

$$\frac{\partial}{\partial t}\left(\mathbf{B} - \delta_L^2\frac{\partial^2 \mathbf{B}}{\partial \mathbf{R}^2}\right) = D\frac{\partial^2 \mathbf{B}}{\partial \mathbf{R}^2}, \qquad D = \nu\delta_L^2 \tag{4}$$

where δ_L is the London penetration depth. The dissipation in this case is related to the skin effect. Since in Eqs. (3) and (4) the dissipation has a diffusion nature, the Langevin sources $\mathbf{J}_{\vec{\Omega}}$ and $\mathbf{J}_\mathbf{B}$ for the flicker noise are given by expressions of the type (1). After averaging over k the equation for $\mathbf{B}$ becomes

$$\left(\frac{\partial}{\partial t}\left(\vec{\Omega} + \frac{e^2 n_s}{mc}\mathbf{B}\right)\right)_\omega + |\omega|\,\vec{\Omega}(\omega) = \mathbf{J}_\Omega(\omega); \quad (\mathbf{J}^2_{\vec{\Omega}})_\omega \propto |\omega|\,\langle \delta\vec{\Omega}_V \delta\vec{\Omega}_V\rangle . \tag{5}$$

The spectral density of the source is given by the expression [cf. Eq. (2)]

$$(\mathbf{J}_{\vec{\Omega}}\mathbf{J}_{\vec{\Omega}})_\omega = 2|\omega|\frac{\pi}{\ln(\tau_{obs}/\tau_D)}\langle \delta\vec{\Omega}_V \delta\vec{\Omega}_V\rangle . \tag{6}$$

We see that the dissipative term and the intensity of the noise are proportional to $|\omega|$. The constant components are $\vec{\Omega}(\omega = \tau_{life}^{-1})$ and $\mathbf{B}(\omega = \tau_{life}^{-1})$. For this definition in (5) and (6) the dissipative term and Langevin source are zero, and Eq. (5) is satisfied when $\vec{\Omega}$ and $\mathbf{B}$ are related by the London equation curl $\mathbf{j} = -(e^2 n_s/mc)\boldsymbol{B}$. Accordingly,

$$\frac{\partial^2 \mathbf{B}}{\partial \mathbf{R}^2} - \frac{1}{\delta_L^2}\mathbf{B} = 0. \tag{7}$$

These equations describe the magnetic-field screening (the Meissner effect) and the direct-current distribution along the cross section of the sample.

Using Eq. (5), we can calculate the low-frequency fluctuations. In the case of a natural flicker sound, when inequalities (2) are satisfied and the observation time is $\tau_{obs} \ll \tau_{life}$, at $\delta\mathbf{B} = 0$ the fluctuation spectrum $\delta\vec{\Omega}$ is given by expression (2), after $\vec{\Omega}$ is substituted for n in it. Maxwell's equations must be used to calculate the spectrum of induced fluctuations (those which are proportional to $\delta\mathbf{B}$).

On the basis of this discussion we can assume that superfluid He^4 can exist in a flow through narrow gaps because of the presence of low-frequency fluctuations of the flow with a $1/f$ spectrum. In this case the diffusion coefficient is on the order of the Planck's constant $\hbar$.

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