Penetration of magnetic field into Josephson structure: the critical state

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The critical state of the Josephson structures is described on the basis of a microscopic method. Several Josephson junctions are used to calculate the critical current J_c vs the induction B, $J_c(B)$.

The penetration of the magnetic field H into a superconductor when the field is higher than the lower critical field is one of the most important problems of superconductivity theory.[1–5] The main objective here is to determine the relationship between the critical current J_c and the induction B which, along with the Maxwell's equation

$$\frac{1}{\mu}\,\mathrm{curl}\,\mathbf{B} = \frac{4\pi}{c}\,\mathbf{J}_c(B), \tag{1}$$

(μ is the permeability), determines the magnetic-field profile. The functional dependence $J_c(B)$ is determined by the flux pinning and is usually defined phenomenologically [$J_c \sim \mathrm{const}$ (Refs. 1 and 2 and $J_c \sim B^{-1}$ (Refs. 3 and 4)].

On the basis of the present understanding of the flux pinning in Josephson structures[6,7] we will describe, by using a microscopic method, the critical state of the Josephson media. In particular, we will obtain the functional dependence $J_c(B)$ and the field profile.

We will use a linear Josephson junction with pinning centers to illustrate the basic idea of this method (Fig. 1). Such centers, for example, are cavities or crossing points of other junctions (a model of a strongly anisotropic lattice[6,7]). Let us consider the penetration of the magnetic field. We assume that the external field H_0 is smaller than

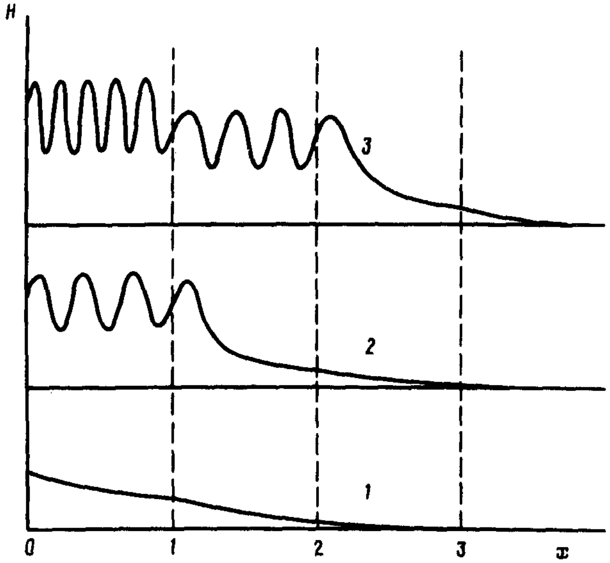

FIG. 1. Schematic diagram of the penetration of the magnetic field into a semi-infinite Josephson junction (along the x axis) for three different fields: $H_1 < H^*_{c1}$ (curve 1), $H_3 > H_2 > H^*_{c1}$ (curves 2 and 3).

the lower crticial field H^*_{c1} and that the field decreases exponentially with distance from the surface (curve 1 in Fig. 1). When $H_0 > H^*_{c1}$, flux begins to penetrate into the junction (the surface barrier is ignored). Because of the pinning, the flux is trapped between the surface and site 1 (curve 2 in Fig. 1). With a further increase of H_0, the flux density n_1 in this part of the junction increases gradually, and so does the pressure of the magnetic force exerted on the pinning center. At a certain value of H_0, when n_1 reaches the maximum permissible value, $n_{\max}$, the magnetic force is equal to the maximum force exerted on the pinning center, a case which corresponds to the critical state. As H_0 continues to increase, the flux penetrates into the second section of the junction and n_2 no longer is equal to zero (curve 3 in Fig. 1). A further increase of H_0 causes the flux to penetrate deeper and deeper into the junction. We can thus formulate a critical state in which a sudden change in the flux density at the neighboring junctions reaches a maximum permissible value which is set by the characteristic features of the pinning centers.

To illustrate this physical model, we will use the example in which the magnetic field penetrates the junction along the [1,1] axis of a Josephson square lattice. The phase difference θ of each junction satisfies the equation $\delta^2\theta'' = \sin\theta$, where δ is the Josephson length.[6–8] The first integral of this equation makes it possible to introduce the invariant

$$\alpha^2 = \frac{1}{2}\left(\frac{1}{2}\delta^2\theta'^2 + \cos\theta - 1\right). \tag{2}$$

In general, α^2 changes from link to link. On the basis of the arguments of the

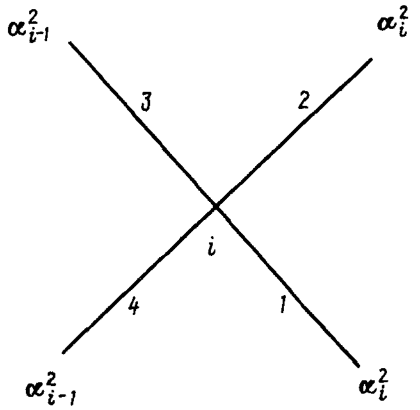

FIG. 2. Labeling of the links which emerge from the lattice site with an index i.

chosen symmetry of the problem, however, it follows that α^2 depends only on the distance from the surface; i.e., the problem reduces to a one-dimensional problem. The vortex concentration n_i in the ith link is related to α_i by the relation

$$n_i = (1+\alpha_i^2)^{1/2}/(2\delta K(1/(1+\alpha_i^2))), \tag{3}$$

where $K(x)$ is the total elliptic integral.[9] In accordance with the recipe described above, let us determine the maximum jump n allowable in the passage through a site in the direction of the field penetration. At each site the matching conditions should be satisfied: $\theta^{(1)}+\theta^{(2)}=\theta^{(3)}+\theta^{(4)}$ and $\theta^{(1)\prime}=\theta^{(2)\prime}=\theta^{(3)\prime}=\theta^{(4)\prime}\equiv\theta'$, where the primes specify the directions of the links which converge on the given site[6,7] (Fig. 2). The symmetry of the problem suggests a pairwise equality of the invariants $\alpha^{(1)}=\alpha^{(2)},\alpha^{(3)}=\alpha^{(4)}$. Taking into account the matching conditions, we find that $\theta^{(2)}=\pm\theta^{(1)}+2\pi m_1$ and $\theta^{(4)}=\pm\theta^{(3)}+2\pi m_2$. Let us consider all possible versions of these equalities. If $\theta^{(2)}=\theta^{(1)}$ and $\theta^{(3)}=\theta^{(4)}$, then from the matching condition we find $\theta^{(1)}=\theta^{(3)}$, and hence $\alpha_i=\alpha_{i-1}$. If $\theta^{(1)}=\theta^{(2)}$, on the other hand, and if $\theta^{(4)}=\theta^{(3)}\pm 2\pi$, we find $\theta^{(3)}=\theta^{(1)}\mp\pi$. It then follows from (2) that

$$\alpha_{i-1}^2=\frac{1}{2}\left(\frac{1}{2}\delta^2\theta_i'^2+\cos\theta_i-1\right),\qquad \alpha_i^2=\frac{1}{2}\left(\frac{1}{2}\delta^2\theta_i'^2+\cos\theta_i^{(1)}-1\right).$$

Taking the difference of these values, we find $\alpha_{i-1}^2-\alpha_i^2=\frac{1}{2}\delta^2\theta_i'^2-2\alpha_i^2-1$. According to (2), the derivative of θ_i' changes in the range $4\alpha_i^2\leqslant\delta^2\theta_i'^2\leqslant 4\alpha_i^2+4$. The maximum jump occurs at the boundary of this interval when $\delta^2\theta_i'^2=4\alpha_i^2+4$. It follows that the critical state is described by the equation

$$\alpha_{i-1}^2-\alpha_i^2=1. \tag{4}$$

The solution of this equation is obvious: $\alpha_i^2=i_0-i$ for $i<i_0$ and $\alpha_i^2=0$ for $i\geqslant i_0$. The value of i_0 is determined from the boundary condition which is shown below. Since α_i is known, we can determine from (3) the vortex concentration in each link. Let us

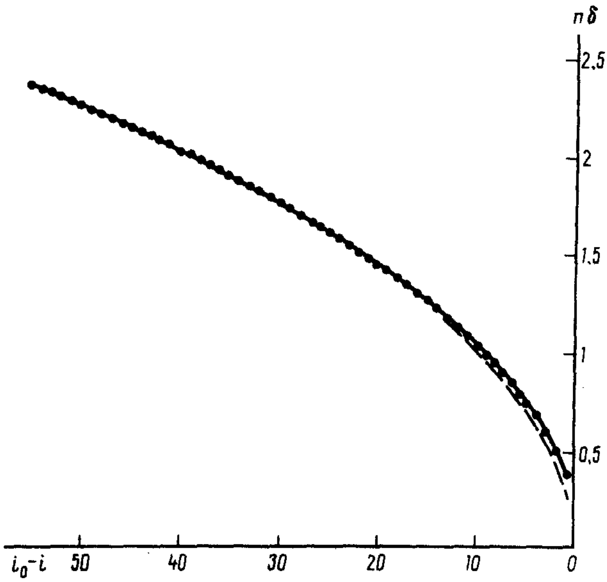

FIG. 3. Vortex concentration n_i vs the number of the link, which penetrates the square lattice along the [1,1] direction, determined by means of numerical calculations based on Eqs. (3) and (4). The dashed curve is the asymptotic dependence $n\delta = \sqrt{i_0 - i/\pi}$.

now find a link α_i with an average magnetic field H_i at the contacts. Each point of the contact satisfies the relation[10] $H = \phi_0\theta'/4\pi\lambda$ (here the intrinsic width of the contact, in contrast with the London length λ, is ignored). Far from the front $i = i_0$, i.e., when $i_0 - i \gg 1$, we have $\alpha_i^2 \gg 1$. In this region the vortex concentration at each link is, according to (3), $n_i = \alpha_i/\pi\delta$. The result of a numerical calculation of the concentration n_i, with allowance for the exact relation (3), is shown in Fig. 3. On the other hand, it follows from (2) that $\theta'^2 \approx 4\alpha^2\delta^{-2}$ and that the average magnetic field in the ith link is $H_i = \phi_0 a_i/2\pi\lambda\delta = \phi_0 n_i/2\lambda = \phi_0\sqrt{i_0 - i}/2\pi\lambda\delta$. It is clear that i_0 is governed by the condition that H_i be equal at the boundary, i.e., at $i = 1$ it is equal to the external magnetic field H_0.

We can now show that Eq. (4) reduces to Eq. (1) in the continuum limit. To this end, let us determine the induction B in each unit cell. It is easy to see that there are $2nL$ vortices in each unit cell L^2 in area, where L is the coupling length. The induction is defined as $B = 2\phi_0 nL/L^2 = 2n/L$. Using the relation $H = \phi_0 n/2\lambda$, we find $B = \mu H$, where $\mu = 4\lambda/L$ is the effective permeability $(L \gg \lambda)$.[6,7,11]

In the region $\alpha_i \gg 1$ the finite difference equation (4) becomes a differential equation in the continuum limit by means of a transformation from a discrete coordinate $x_i = \Delta i$, where $\Delta = L/\sqrt{2}$, to a continuous x. Since $H_i \sim \alpha_i$ and $B_i = \mu H_i$, the differential equation which we obtained takes the form (1) with $J_c(B) = j_c\tilde{B}/B$, where j_c is the critical current of the Josephson junction[10] and $\tilde{B} \equiv \mu\phi_0/(\sqrt{2}\pi L\lambda)$. This result corresponds to the Anderson and Kim's theory.[3,4]

Another case worth mentioning is the penetration of the magnetic field into a

Josephson linear junction which is crossed by short Josephson junctions of length $2l \ll \delta$. These junctions, which are situated a distance L from each other, are the pinning centers.[6,7] The method which we developed above can be used to show that the critical state is described by Eq. (1) with $\mu = 1$ and that $J_c(B) = j_c B_1/(B + B_0)$, where $B_0 = \phi_0/4\pi l\lambda$, and $B_1 = \phi_0/2\pi L\lambda$. In fields $B \ll B_0$ we thus have $J_c \approx j_c B_1/B_0$, and the field profile is described by the Bean theory.[1,2] In strong fields $B \gg B_0$ we have $J_c(B) \approx j_c B_1/B$, in agreement with the theory of Anderson and Kim.[3,4]

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