Spontaneous parity violation in three-dimensional scalar electrodynamics

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It is found that in a (2 + 1)-dimensional Abelian Higgs system radiation corrections include spontaneous parity violation. The bare topological term evolves to a fixed value that depends on the sign but not the magnitude of the bare coefficient.

Gauge theories in 2 + 1 dimensions can contain in their action a topological term that breaks P and T symmetry.[1] This term arises, for instance, as a result of radiation corrections to the electrodynamics of massive fermions.[2] It has recently become popular to include topological Lagrangians in the context of high-T_c superconductivity.[3–5] If the topological action is responsible for the superconductivity, then at least a part of it must appear during the metallization and be associated with the low-lying excitations that transport charge in the metallic (superconducting) phase. These excitations may be Cooper pairs or the hypothetical zero-spin bosons (holons) of the resonating valence bond theory. The question arises as to whether scalar particles can induce the topological term.

In contrast to the electrodynamics of fermions, whose mass violates P and T symmetry, the scalar theory does not contain explicit parity violation. We shall show, however, that if we introduce into the Lagrangian the bare topological term

$$L_{CS}=\frac{\kappa_0}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda, \tag{1}$$

the same structure arises in the single-loop approximation, but with $\varkappa_0\rightarrow\varkappa$, where, in the strong coupling limit

$$\kappa=\frac{2}{3\pi}\frac{\kappa_0}{|\kappa_0|}\;. \tag{2}$$

[compare with the contribution of the massive fermion:[2] $\varkappa_F=(1/4\pi)m/|m|$]. This expression does not depend on the magnitude of the bare coefficient and indicates spontaneous parity violation.

Let us consider a (2 + 1)-dimensional Abelian Higgs system with the topological term (1):

$$L=-\frac{1}{4e^2}F^2_{\mu\nu}+\frac{\kappa_0}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+|D_\mu\Phi|^2-\lambda(\Phi^*\Phi-c^2)^2+L_{gf}, \tag{3}$$

where $D_\mu=\partial_\mu-iA_\mu$, and L_{gf} fixes the gauge (it may contain ghosts). The structure of interest to us in the vector propagator does not depend on the gauge and is

FIG. 1

$$D_{\mu\nu}(k) = e^4\kappa_0\epsilon^{\mu\nu\lambda}\frac{k_\lambda}{(m_W^2-k^2)^2-\kappa_0^2e^4k^2}. \tag{4}$$

In the single-loop effective action the contribution proportional to the ε-tensor comes from the diagram in Fig. 1 (the vertices are proportional to the scalar condensate). Introducing the coefficient $\varkappa$, analogously to $\varkappa_0$ in (4) we write the result of the calculations as

$$\kappa = \frac{4e^2}{3\pi^2}m_W^2\kappa_0 I(\kappa_0),\quad I(\kappa_0)=\int_0^\infty dq\,\frac{q^4}{(m_H^2+q^2)^2((m_W^2+q^2)^2+\kappa_0^2e^4q^2)}, \tag{5}$$

where $m_W=\sqrt{2}ec$, and m_H is the mass of the Higgs boson.

If e^2 is finite, then as $\varkappa_0\to 0$, the integral in (5) approaches a finite value, and consequently, $\varkappa\to 0$. An instability arises in the limit $e^2\to\infty$, i.e., in a theory without a tree kinetic term $\propto F_{\mu\nu}^2$. Now the topological term is the leading term and may not be harmlessly discarded. For $\varkappa_0\to 0$

$$I(\kappa_0)=\int_0^\infty dq\,\frac{1}{m_W^2+\kappa_0^2e^4q^2}=\frac{\pi}{2}\,\frac{1}{m_W^2e^2|\kappa_0|}, \tag{6}$$

from which follows (2). It should be noted that this solution is found in the region of strong coupling: from $e^2\gg m_W$ it follows that $e\gg c$. Nonetheless, it is correct by virtue of the nonrenormalization theorems,[6] which state that in the absence of massless fields the material of the highest loops do not give a contribution proportional to the ε-tensor. In our case in the R_ξ gauge, for $\xi\neq 0$ there is not a single massless line. Gauge theory without the tree kinetic term appears unusual (although the CP model is an example), but it is in just this form that it appears in the high-T_c models (see, e.g., Ref. 3 and 4). An important point is that this theory, (including the topological term) is renormalizable. In contrast to, let us say, $(3+1)$-dimensional quantum electrodynamics, the radiation corrections do not give diverging terms proportional to $F_{\mu\nu}^2$. Finite contributions of this form arise from the loops, but even they are not able to eliminate the instabilities on account of these same theorems.[6]

We have thus shown that system (3) for $e^2\to\infty$ is unstable against spontaneous generation of topological action. It would be of interest to know if a similar situation holds in CP^1 models that are used in the description of two-dimensional antiferromagnets.

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