Fermion pair exponentiation in QED[1)]

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(Submitted 25 December 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **51**, No. 2, 72–74 (25 January 1990)

Electromagnetic radiative corrections due to initial state pair production: $e^+ e^-$ annihilation are examined. A new result shows that fermion pair production contributions do exponentiate. This exponent has an infrared behavior that differs from the known soft-photon summation formula.

We have recently investigated[1] the problem of the exponentiation of large perturbative contributions at the edge of the phase space in QCD. For large values of $\tau = Q^2/s$ in the Drell-Yan process, for example, the perturbative terms are accompanied by logarithmic factors and must be resumed[2] in order to get meaningful theoretical predictions. The dynamics associated with this kinematical region corresponds to the dominance of the soft and collinear radiation and is characterized by the presence of double logarithmic contributions. In order to describe and compute the structure of those perturbative terms at the leading and next-to-leading level a technique based on the eikonal approximation combined with evolution equations for structure functions has been developed.[1] As a result, the leading and next-to-leading logarithms have been resummed and the comparison with a complete finite order result at two-loop level[3] has shown complete agreement.

It is known[4] that at the Z^0 resonance the soft and collinear dynamics plays a crucial role in describing the shape of the resonance peak. Structure-function techniques[4] and ordinary Feynman diagram calculations[5] have been used to compute electromagnetic radiative corrections to the lineshape. The physical motivation is the accurate determination, according to the precise measurements, of the parameters of the Z^0 boson in the LEP/SLC experiments.[4,5] The contributions from soft and collinear

 0021-3640/90/020083-04$01.00

photon radiation have been shown to appreciably affect the position and shape of the peak.

The techniques developed in Ref. 1 can been applied to study this problem with an independent method. This approach allows us to evaluate the structure of the non-dominant logarithmic terms which arise from the mass singularities.[1] The $e^+ e^-$ annihilation cross section can be seen as an electrodynamic Drell-Yan process, where the electrons, which are surrounded by the photon radiation, play the role of the annihilating partons. The cross section for $e^+ e^-$ annihilation is

$$\frac{d\sigma}{ds'} = \frac{1}{s}\sigma_0(s')W(s,\tau), \tag{1}$$

where s is the invariant energy square, $\tau = s'/s$, and $W(s,\tau)$ is the radiator which contains radiation contributions to the elementary Born cross section $\sigma_0(s')$. The total cross section is given by $\sigma = \int ds' d\sigma/ds'$.

The perturbative expansion for $W(s,\tau)$ shows contributions of the type

$$\frac{\alpha^n}{(1-\tau)}\ln^{k-1}(1-\tau)L^n, \quad (m+k \leqslant 2n),$$

where $L = \ln(s/m_e^2)$, m_e is the electron mass, and terms of the form

$$\alpha^n L^m f(1-\tau), \quad (m \leqslant n)$$

with the function $f(1-\tau)$ which is integrable in the limit $\tau \to 1$. The contributions of these two types can be classified as collinear-soft and non-soft, respectively.

To perform the resummation of the logarithmic contributions, it is useful to consider instead of the $\tau \to 1$ limit, the large N behavior of the N-moment distributions, $W_N(s) = \int d\tau \tau^{N-1} W(s,\tau)$. After conservation of energy has been taken into account, which can be done in a natural way in the expression of the nth moment, the "radiator" $W_N(s)$ becomes[7]

$$\ln W_N^{IR}(s) = -\int \frac{d^3q}{4\pi\omega_q}\left(\frac{p_1}{p_1\cdot q} - \frac{p_2}{p_2\cdot q}\right)^2 A\,[\alpha(q_t^2)] \times\left[\left(1 - 2\frac{\omega_q}{\sqrt{s}}\right)^{N-1} - 1\right]\theta(\sqrt{s}/2 - \omega_q), \tag{2}$$

where $\alpha(q^2)$ is the QED running coupling constant, p_1 and p_2 are the electron and positron momenta, and the superscript IR means that Eq. (2) takes into account the leading and the next-to-leading collinear-soft contributions.

When only the radiation of photons from the interacting leptons is considered, we have $A(\alpha) = \alpha/\pi$, where α is the fine-structure constant. In this case Eq. (2) gives the usual result[4,6] of the soft-photon exponentiation.

When the production of real and virtual fermion pairs of mass m_f^2 is taken into account, we have

$$A\,[\alpha(q^2)] = \frac{\alpha}{\pi}\left[1 + O\left(\alpha\frac{q^2}{m_f^2}\right)\right], \quad q^2 \leqslant m_f^2 \tag{3a}$$

$$A\,[\alpha(q^2)] = \frac{\alpha(q^2)}{\pi}\left(1 + K_{QED}\,\frac{\sigma(q^2)}{2\pi}\right),\quad q^2 \gg m_f^2, \tag{3b}$$

where $\alpha(q^2) = \alpha/(1 - \alpha/3\pi \ln q^2/m_f^2)(q^2 \gg m_f^2)$, and in the QED case we have $K_{QED} = -10/9$ for the contribution of each charged lepton pair. Inserting expressions (3) into (2), we obtain

$$\ln W_N^{IR}(s) = \frac{2}{\pi}\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\Big[\alpha(L-1) + \theta\,[(1-z)^2 s - m_f^2]$$
$$\times \int_{m_f^2}^{s(1-z)^2} \frac{dq^2}{q^2}\left(\alpha(q^2) - \alpha - \frac{5}{9}\,\frac{\alpha^2(q^2)}{\pi}\right)\Big]. \tag{4}$$

The inverse Mellin transform of the first term in the square brackets give the well-known soft photon exponentiation factor $\beta(1-\tau)^{\beta-1}$ $[\beta = 2\frac{\alpha}{\pi}(L-1)]$ for $W(s,\tau)$. The remaining terms prove that the pair production contributions do exponentiate. At finite order the coefficients of these terms coincide with those given in Ref. 5, since by expanding them up to the second order in α we obtain

$$\left(\frac{\alpha}{\pi}\right)^2 \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\,\theta\,[(1-z)^2 s - m_f^2]$$
$$\times\left[\frac{1}{3}\ln^2\frac{(1-z)^2 s}{m_f^2} - \frac{10}{9}\ln\frac{(1-z)^2 s}{m_f^2}\right] + O(\alpha^3). \tag{5}$$

We wish to emphasize that the pair production contribution differs qualitatively from the pure photonic first term because of an explicit logarithmic $[\alpha\ln(1-z)]^n$ dependence that cannot be reproduced by the ordinary renormalization-group equations.[4,5]

The use of the factorization theorem[7] to allow for the leading collinear terms that are not infrared singularities gives the following additional contribution to the nonsinglet channel:

$$\ln W_N^{non\,IR}(s) = -\frac{1}{\pi}\int_0^1 dz(z^{N-1}-1)(1+z)\int_{m_e^2}^{s}\frac{dq^2}{q^2}\,\alpha(q^2). \tag{6}$$

The total radiator

$$W_N = W_N^{IR}\;W_N^{non\,IR} \tag{7}$$

takes into account all contributions of the type $\alpha^n L^m \ln^k N$ with $m + k \geqslant n$. the coefficients of the terms in the total radiator in Eq. (7) reproduce the finite-order results.[4,5]

A thorough exposition of these results and numerical estimates will be presented in a more detailed publication.

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