Time-dependent holography of the two-photon effect

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The recording of the two-phonon effect, which is excited by spectrally resolved radiation, makes it possible to reconstruct the temporal characteristics of a complex light signal.

As is well known, the temporal resolution provided by electron-optical recording of short-duration light signals does not meet present-day needs. A significantly higher degree of temporal resolution than that achieved by the use of electronics can be achieved on the basis of nonlinear effects in crossed or counteraligned light beams.[1-4] The most widespread methods of such type record the autocorrelation function of the signal intensity and require the use of additional information on the signal in order to completely reconstruct its characteristics. The situation is greatly simplified if it is known beforehand that the radiation consists of one smooth pulse whose duration is to be determined. In general, data on the second-order autocorrelation are not sufficient for a unique reconstruction of the signal. Cross-correlation measurements—in which a convolution of the investigated radiation with a short reference pulse is recorded—turn out to be more informative.[5] However, the domain of applicability of such measurements is limited as a result of the difficulties associated with the formation of the reference pulse, which must be significantly shorter than the signal.

The purpose of the present note is to lay out the principle behind measurements for which a reference pulse is not needed and which do not require the application of *a priori* information on the radiation. The measurements are based on the prior spectral resolution of the investigated radiation $E(t) = \mathrm{Re}\mathscr{E}_S(t)\exp(-i\omega_S t)$, after which it is directed into the nonlinear medium (e.g., into a medium in which the second optical harmonic is generated). In this connection, there is an analogy with Ref. 6. In contrast with Ref. 6, however, here we require accurate measurements of the energetic characteristics of the nonlinear effect, which must be carried out at two values of the spectral resolution: in the sharp image plane of the spectrum and in a defocused image plane. The method is based on the fact that the efficiency of two-photon conversion depends on the phase relations of the spectral components of the exciting radiation.

Let us consider the field $\varepsilon(\omega,t,\Gamma)$ in the exit plane of the spectral device. Let its connection with the Fourier spectrum of the investigated signal, i.e., with $\tilde{\mathscr{E}}(\omega') = \int \mathscr{E}_S(t)\exp(-i\omega_S t + i\omega' t)dt$, have the form

$$\mathscr{E}(\omega, t, \Gamma) = \frac{1}{\sqrt{\pi}\Gamma} \int \tilde{\mathscr{E}}(\omega') \exp[-i\omega' t - (\omega' - \omega)^2/\Gamma^2] d\omega'. \tag{1}$$

Here ω is the current frequency which is uniquely associated with the coordinate in the exit plane of the spectral device ($\omega = Dx$, where D is the dispersion), and $\Gamma = \gamma_0$, where γ_0 is the width of the apparatus function of the device. We assume that the field in the defocused image plane is also expressed in terms of the initial signal according to Eq. (1), with the modification that there also enters into the formula a second spectral width: $\Gamma = \gamma$, $\gamma > \gamma_0$. We assume that the width of the instrumental function γ_0 satisfies the inequality

$$\gamma_0 \ll 1/T, \tag{2}$$

where T is the total duration of the signal, $1/T$ is the characteristic scale of variation of the signal spectrum, and γ satisfies the inequality

$$\gamma \lesssim 1/T, \tag{3}$$

We will find for these two cases the value of the energy U which is liberated during two-photon conversion

$$U(\omega, \Gamma) = \chi \int | \mathscr{E}(\omega, t, \Gamma)|^4 dt \tag{4}$$

(here the coefficient χ incorporates all the necessary characteristics of the nonlinear medium). We now set $\Gamma = \gamma$ in Eq. (1). Making use of condition (3), we expand the complex amplitude $\tilde{\mathscr{E}}(\omega)$ in a series in the frequency for $|\omega' - \omega| \lesssim \gamma$. We assume for simplicity of the calculations that the modulus of the function $\tilde{\mathscr{E}}(\omega')$ varies much slower than its phase, and we write

$$\tilde{\mathscr{E}}(\omega')|_{|\omega' - \omega| \sim \gamma} = \tilde{\mathscr{E}}(\omega)\exp[i\Phi(\omega')] = \tilde{\mathscr{E}}(\omega)\exp[ia(\omega' - \omega) + ib(\omega' - \omega)^2]. \tag{5}$$

Substituting Eq. (5) in Eq. (1), we find

$$\mathscr{E}(\omega, t, \gamma) = \tilde{\mathscr{E}}(\omega)\gamma^{1/2}(1 - ib\gamma^2)^{-1/2}\exp[-i\omega t - \frac{(t-a)^2\gamma^2}{4(1 - ib\gamma^2)}]. \tag{6}$$

From Eqs. (6) and (4) we obtain the energy liberated in the two-photon effect

$$U(\omega, \gamma) = \pi^{1/2}\chi\gamma(1 + b^2\gamma^4)^{-1/2} | \tilde{\mathscr{E}}(\omega)|^4. \tag{7}$$

Analogous expressions also hold for $\Gamma = \gamma_0$; however, taking inequality (2) into account, now, we ignore the quantity $b\gamma_0^2$ in comparison with unity and write

$$U(\omega, \gamma_0) = \pi^{1/2}\chi\gamma_0 | \tilde{\mathscr{E}}(\omega)|^4. \tag{8}$$

From expression (7) it is clear that the magnitude of the two-photon effect depends on the variation of the phase in an isolated region of the spectrum, i.e., on $b(\omega) = d^2\Phi/d\omega^2(\omega)$. The quantity $|b(\omega)|$ can be found from the quantities $U(\omega,\gamma)$ and $U(\omega,\gamma_0)$ [Eqs. (7) and (8)]:

$$|b(\omega)| = \gamma^{-2}\{(\gamma/\gamma_0)^2[U(\omega,\gamma_0)/U(\omega,\gamma)]^2 - 1\}^{1/2}. \tag{9}$$

After determining the sign of $b(\omega)$ from the condition of continuity of the higher derivatives of the phase, and after finding the quantity $|b(\omega)|$, we can determine the phase of the spectrum by integration

$$\Phi(\omega) = \int_{\omega_S}^{\omega} d\omega' \int_{\omega_S}^{\omega'} b(\omega'')d\omega''. \tag{10}$$

Here we have set $d\Phi/d\omega(\omega_S) = \Phi(\omega_S) = 0$ (these constants have no effect on the shape of the signal and can be chosen arbitrarily). The complex function $\tilde{\varepsilon}(\omega)$ is thus completely reconstructed, which means that the complex signal itself, which is related to $\tilde{\mathscr{E}}(\omega)$ by the Fourier transformation, is also completely reconstructed.

We emphasize that the proposed method enables one to obtain the time course of the phase as well as the intensity of the field, and in this regard is equivalent to time-dependent holography.[7,8] It is important that in contrast with the well-known methods of time-dependent holography or cross-correlation measurements, the proposed method does not require the use of a short reference pulse. The most serious requirement of this method is for a high enough level of accuracy in the measurement of the yield of the two-photon effect to allow the recording of the difference of $\sqrt{1+b^2\gamma^2}$ from 1 for $|b|\gamma^2 \sim 1/4$–$1/2$. The realization of such measurements would undoubtedly be a step forward in the study of short duration light signals.

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