Josephson transition with nonlocal interaction

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The Josephson effect in thin films ($d \ll \lambda$) is analyzed. A nonlocal equation for a phase in the case of extended contacts is derived. The asymptotic characteristics of vortices in such films are studied.

Many theoretical studies of the Josephson effect have focused on tunnel junctions consisting of bulk superconductors of thickness $d_{1,2} \gg \lambda_L$ (where λ_L is the London penetration depth) or tunnel junctions consisting of superconductors of finite widths, $d_{1,2} \lesssim \lambda_L$, under the assumption that electrodynamics of superconductors themselves can be described by the London equations[1] (see also Refs. 2 and 3). However, even when films have finite dimensions, the theoretically considered configuration of the tunnel junction is such that the dimension along the magnetic field is infinite. This circumstance, along with the local basic equations, accounts for the local nature of the equation describing the space-time evolution of the phase difference. Experimentally, however, contacts whose dimension directed along the field is finite and commensurate with the London penetration depth λ_L, are used frequently. A configuration of this sort of found, for example, in Y–Ba–Cu–O single-crystal wafers with twins whose wafer thickness may be on the order of λ_L.

We will show that in tunnel junctions consisting of thin superconducting films ($d \ll \lambda_L$) the current link j with a phase difference φ is nonlocal in nature. This circumstance accounts for the power-law decay of the magnetic field and current of a vortex at large distances from its center.

To simplify the computations, we will consider a steady-state case and assume that the superconducting films that form the contact are the same. The geometry which we chose is shown in Fig. 1. This film can be regarded as a plane ($z = 0$) along which a current flows, $\mathbf{j}\delta(z)$. For a planar current the following equation is valid for each superconductor:

$$\frac{4\pi}{c}\mathbf{j}_{1,2} = \text{curl curl } \mathbf{A} = -\Delta \mathbf{A} = \frac{1}{\lambda_e}(\Phi_{1,2} - \mathbf{A})\delta(z), \tag{1}$$

where $\Phi_{1,2} = (\varphi_0/2\pi)\widehat{\nabla}\varphi_{1,2}$ and $\lambda_e = \lambda_L^2/d$, is the effective penetration depth for a thin plate in a field which is parallel to its plane.[4] The boundary conditions of Eq. (1) at the edges of the Josephson junction ($y = 0$) are

$$-\Delta A_y \big|_{y=0} = \frac{4\pi}{c} j_y(x, 0). \tag{2}$$

On the other hand, the current that passes through the junction, $j_y(x)$, is related to the phase difference $\varphi = \varphi_1 - \varphi_2$ by

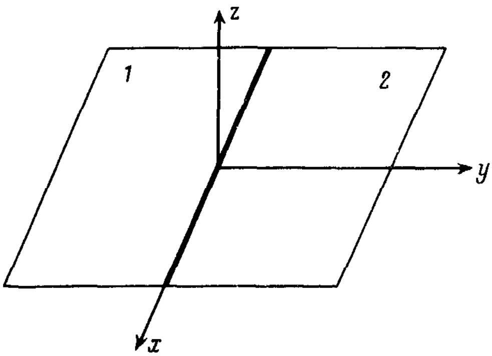

FIG. 1.

$$j_y(x) = j_c \sin\varphi \;, \tag{3}$$

where j_c is the critical current through the junction. Transforming expression (1) with allowance for (2) and (3), we obtain an equation for the phase difference $\varphi(x)$

$$\frac{\phi_0}{\pi^3\lambda_e}\frac{\partial^2}{\partial x^2}\int_{-\infty}^{\infty} dp\, e^{ipx/\lambda_e}\varphi(p)J(p) = \frac{4\pi}{c} j_c \sin\varphi \,. \tag{4}$$

Converting to dimensionless variables $x \rightarrow x/\tilde{\lambda}_j$, we find

$$\frac{\partial^2}{\partial x^2}\int \varphi(x')\, \mathfrak{F}(x - x')dx' = \sin\varphi \,, \tag{4'}$$

where $\tilde{\lambda}_j = \lambda_j\sqrt{d/\lambda_L}$, λ_j is the Josephson penetration depth,

$$\mathfrak{F}(y) = \frac{2\lambda_e}{\pi^2\tilde{\lambda}_j}\int J\left(p\frac{\lambda_e}{\tilde{\lambda}_j}\right)e^{ipy}\,dp \,,$$
$$J(p) = \frac{2}{\sqrt{4p^2-1}}\arctan\frac{\sqrt{4p^2-1}}{1+2|p|} \tag{5}$$

We will use Eq. (4) below in the form

$$\frac{\partial}{\partial x}\int_{-\infty}^{\infty}\varphi'(x')\,\mathfrak{F}(x-x')\,dx' = \sin\varphi \tag{6}$$

To determine the type of single vortex, we must solve Eq. (6) in which the condition $\varphi(x=\infty) - \varphi(x=-\infty) = 2\pi$ is satisfied. Clearly, such a solution always exists and its asymptotic behavior is governed by the function $\mathfrak{F}(x-x')$ at large values of x. A simple analysis shows that in the limit $x \rightarrow -\infty$ we have

$$\varphi \approx 2\pi \frac{\partial}{\partial x} \mathfrak{F}(x) \approx 4\lambda_e / \pi \tilde{\lambda}_j x^2 . \tag{7}$$

Accordingly, in the limit $x \to \infty$ we have $\varphi \approx 2\pi - 4\lambda_e / \pi \tilde{\lambda}_j x^2$. Far from the vortex center the magnetic field falls off as $1/x$, i.e., much slower than the fields of the Abrikosov vortices in thin films.[4] At the vortex center the current is zero, while the filed reaches a maximum value. At short range the structure of the Josephson vortex in films is qualitatively the same as that in the bulk samples. At a nonzero potential difference at the barrier the equation for the phase φ is

$$\frac{\partial^2}{\partial x^2} \int_{-\infty}^{\infty} \varphi(x') \mathfrak{F}(x - x') dx' - \frac{\partial^2 \varphi}{\partial t^2} = \sin\varphi . \tag{8}$$

In contrast with the equation which describes the Josephson effect for a standard configuration, Eq. (8) does not have a Lorentz-invariant form. This circumstance leads, in particular, to a disruption of the proportionality between the motion of a vortex in the junction and the electric field strength, which manifests itself in the change of the I-V characteristic compared with that of a junction which consists of bulk superconductors.

Since the function $\mathfrak{F}$ diverges in the limit $|x - x'| \to 0$, the energy of the tunnel junction can be written in the form

$$\mathcal{E} = \frac{\hbar j_c}{2e} \int_{-\infty}^{\infty} dx \{ 1 - \cos\varphi(x) + \frac{\varphi_t^2}{2}$$
$$+ \frac{\varphi'(x)}{2} \int_{-\infty}^{\infty} \varphi'(x') \mathfrak{F}(x - x') dx' \} \approx \frac{\hbar j_c}{2e} \{ \int_{-\infty}^{\infty} dx \, (1$$
$$- \cos\varphi(x) + \frac{\varphi_t^2}{2} + C(a) \varphi'^2(x))$$
$$+ \int_{-\infty}^{\infty} dx \varphi'(x) \int_{a}^{\infty} (\varphi'(x - x') + \varphi'(x - x')) \mathfrak{F}(x') dx' \} . \tag{9}$$

Here a is the cutoff parameter, and $C(a) \equiv \frac{1}{2} \int_{-a}^{a} \mathfrak{F}(y) dy < \infty$. It is easy to see that all the integrals in (9) diverge and, hence, the energy corresponding to a solitary static vortex is finite and is a function of the parameter.

We will estimate the energy of two static vortices which are spaced widely apart. Let us assume that the distance between the vortices is $x_1 - x_2 = \Delta x \gg 1$. We can then write an asymptotic expression for a two-vortex solution in the form

$$\varphi(x) \approx \varphi_1(x - x_1) + \varphi_2(x - x_2). \tag{10}$$

Substituting solution (10) in the expression for the energy, we find

$$E = E_1 + E_2 + E_{int} ,$$

where E_1 and E_2 are the energies of the first and second vortices, respectively, and E_{int} can be interpreted as the potential energy of the static vortices

$$E_{int} \approx 8\sigma_1\sigma_2\lambda_c/\tilde{\lambda}_j|\Delta x|,$$

where $\sigma = 1$ for a vortex, and $\sigma = -1$ for an antivortex. Since the effect of kinks on each other distorts their shape, we estimate the additional contribution to the interaction energy to be $\Delta E_{int} \sim \varphi'(\Delta\tilde{x}) \sim (\Delta x)^{-3}$. Such a slow decay of the vortex interaction gives rise to the appearance of a gap in the vortex lattice vibration spectrum. As a result, at large separations the correlation function of the phase will not diverge logarithmically[5] but rather will be a bounded function. In other words, the nonlocal interaction in Eq. (4) in fact reconstructs the long-range order in the vortex system.

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