Dedicated processor for studying Ising model on random lattice

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A processor has been developed with an architecture which reflects the structure of the Monte Carlo method for the Ising model with random links on a lattice containing 256×256 spins. The parallel implementation of operations makes it possible to achieve a speed of more than 4×10^6 elementary Monte Carlo steps per second.

In this letter we are reporting the development of a dedicated computer which operates at a speed comparable to that of a supercomputer but whose cost is only a small fraction of that of an ordinary personal computer.

Our computer is designed for studying the critical behavior of the Ising model on

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a lattice with random links. This problem has been studied intensely, by both analytic and numerical methods. Corresponding bibliographies are given in Refs. 1 and 2. The different theories lead to contradictory results. Numerical studies by the Monte Carlo method[3] on general-purpose computers require a substantial amount of time, so we do not yet have reliable results.

In the Ising model, the energy of a single spin which is interacting with its neighbors is

$$\mathcal{E} = -\sigma_{\mathbf{x}} \sum_{\vec{\mu}} J_{\mathbf{x},\vec{\mu}} \sigma_{\mathbf{x}+\vec{\mu}}, \tag{1}$$

where x is the coordinate of spin σ_x on the lattice, $x + \vec{\mu}$ are the coordinates of the four neighboring spins $\sigma_{x,\vec{\mu}}$, and $J_{x,\vec{\mu}}$ is the exchange integral. In our case, the exchange integral can take on two arbitrary values on each link.We denote these values by J and J'.

The elementary step of the Monte Carlo process is as follows: A site is selected at random on a square lattice. The energy in (1) is calculated at this site, and the probability for a flip of the central spin, P, is also calculated:

$$P = \exp(\beta\mathcal{E})/(\exp(\beta\mathcal{E}) + \exp(-\beta\mathcal{E})), \tag{2}$$

where β is the reciprocal temperature. The probability P is compared with a random number between 0 and 1. If P is greater than this random number, then the spin at this site flips. This completes the Monte Carlo step, and it is time to select another random node.

For our problem it is thus necessary to carry out some fairly simple operations repeatedly. It is in such cases that dedicated processors help make it possible to solve the problem in a reasonable time.

The purposes of dedicated processors and problems in designing them are described in the review of Herrmann.[4] Our dedicated processor has approximately the same structure as that of the dedicated processor at Delft[5] but is much simpler.

Figure 1 shows a block diagram of the processor. We will follow this diagram in describing the operation of the processor.

The processor operates under the control of a control computer, in our case an IBM PC/XT (we will say simply PC). The PC stores the values of the links and the initial values of the spins in the main memory of the processor. The main memory of the dedicated processor consists of nine-bit words. These nine bits describe a lattice-site (Fig. 2): the value of the central spin, $\sigma_x = +1, -1$ (the bit will contain 0 or 1, respectively); the values of its four neighbors; and also information about the four exchange integrals $J_{x,\vec{\mu}}$ (the values J and J' correspond to the bit value 0 and 1). The PC then stores 500 16-bit random numbers in the dedicated processor for an initialization of two random-number generators. These generators are identical and are designed on the basis of an algorithm which leads to good statistical properties for the class of problems of interest here[6] and which has a long random-sequence period (2^{250}): $x_{n+250} = x_n \oplus x_{n+103}$, where x_n are integers, and $\oplus$ means the binary Boolean operation "excluding OR." The PC then stores 16-bit values of the energies and of the

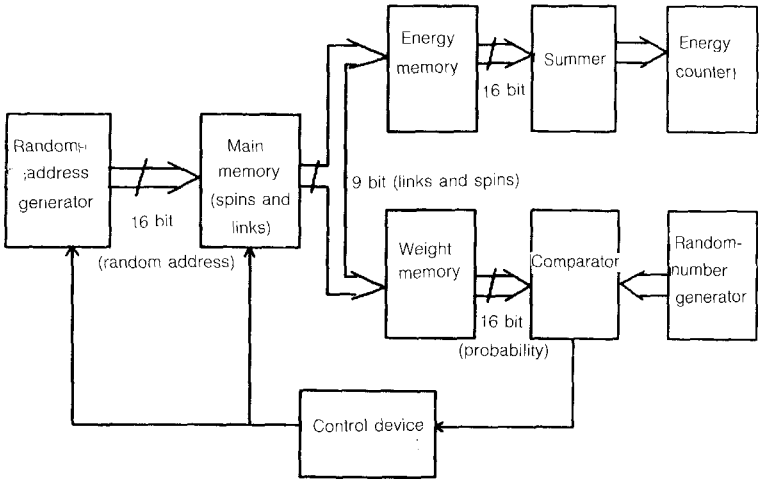

FIG. 1. Block diagram of the processor.

spin-flip probabilities in the energy memory and in the weight memory for all 2^9 possible local configurations. After the counters are reset, a signal to begin operation is generated.

An elementary Monte Carlo step contains the following: a determination of the index of the site by generator of a random address, a reading of this site from the main memory, a reading of the spin-flip probabilities from the weight memory (nine bits of a site constitute the address of a probability in this memory), the generation of a random number by the random number generator, a comparison of this number with the flip probability, and a storage (or nonstorage) of the flipped spin at five addresses

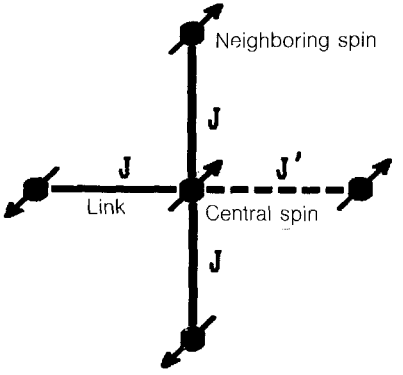

FIG. 2. Configuration of a site.

in the main memory (the address of the given site and the addresses of its four neighbors).

The reading from the main memory of the dedicated processor is carried out in parallel with all other operations. The flipping of a spin requires one additional main-memory cycle for storage. The average probability for the flipping of a spin near the point of a phase transition is about 1/6. The elementary Monte Carlo step thus lasts 16% longer, on the average, than one memory cycle. Refreshing of the dynamic memory takes a few percent more of the time. We used integrated circuits with a 200-ns memory cycle and a 90-ns access time. An average of less than 250 ns is thus expended on a single Monte Carlo step, and it is this time which determines the speed of the processor. A dedicated processor of the same design using advanced memory integrated circuits might be faster by a factor of 10.

The dedicated processor simultaneously calculates the energy and moment of the spin system. For this purpose, it contains an accumulating summer for the energies of the sites. In order to find the total energy and the total spin of the entire lattice, we used 34-bit counters, whose values are read by the control computer. In taking this approach, we can thus also directly calculate the specific heat and spin susceptibility.

Let us compare the results for the pure Ising model obtained on a 128×128 lattice with the exact solution for two temperatures, above and below the point of a phase transition:

$\beta \cdot J$	E_{exact}	E	DE
0,3	0,35225	0,35225	$8 \cdot 10^{-5}$
0,6	0,95454	0,95453	$1 \cdot 10^{-5}$

Here E is the reduced energy at a site, and DE is the relative standard deviation. We hope to obtain new results for an Ising model with random links in the near future.

The dedicated processor described above could also be used to study other problems, e.g., the Ising model in a random magnetic field, a 2D spin glass, and the percolation problem.

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