Penetration of inclined vortices into layered superconductors

A. I. Buzdin and A. Yu. Simonov

Institute of High-Pressure Physics, Academy of Sciences of the USSR; M. V. Lomonosov Moscow State University

(Submitted 10 January 1990)

Pis'ma Zh. Eksp. Teor. Fiz. **51**, No. 3, 168–171 (10 February 1990)

The penetration of an oblique field into layered anisotropic superconductors at $H = H_{c1}$ should occur not in the form of distinct filaments but as chains of vortices. The equilibrium periods of these chains are found for various orientations of the field and for various degrees of anisotropy.

A distinctive feature of inclined Abrikosov vortices into layered superconductors is that the screening currents flow primarily in the plane of the layers, not in the plane perpendicular to the axis of a vortex, as in ordinary isotropic superconductors.

For our analysis we work from a Ginzburg–Landau functional with an anisotropic effective mass (Ref. 1, for example):

 0021-3640/90/030191-05$01.00

$$G = a|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{1}{4m_\mu}|(\hbar\nabla_\mu - \frac{2ie}{c}A_\mu)\Psi|^2 + \frac{(B-H)^2}{8\pi}, \quad (1)$$

where the coordinate system has been chosen in such a way that the z axis runs along the normal to the layers, $\vec{\nu}$, and $m_i = (m_X, m_Y, m_Z)$ are the principal values of the "inverse effective mass" tensor, where $m_X = m_Y = m_\parallel < m_Z = m_\perp$.

To analyze the field of a vortex, it is convenient to use an orthogonal coordinate system with $\mathbf{z}'$ axis running along the vortex axis $\mathbf{l}$, with $\mathbf{y}'$ axis coinciding with the original $\mathbf{y}$ axis [see (1)], and with a new $\mathbf{x}'$ axis lying in the $(\vec{\nu},\mathbf{l})$ plane. At this point, we drop the primes. In the Fourier representation, the problem of the distribution of the magnetic field $\mathbf{h}_k$ of an inclined vortex is known to have an exact solution,[2-4] and we have

$$\mathbf{lh}_\mathbf{k} = \frac{\Phi_0}{\lambda^2} \frac{\lambda^{-2} + (q^2 + Q^2)(1 + \epsilon\nu_z^2)}{(\lambda^{-2} + q^2 + Q^2)[\lambda^{-2} + (1+\epsilon)q^2 + (1+\epsilon\nu_z^2)Q^2]}, \quad (2)$$

where $\nu_z^2 = (\vec{\nu}\mathbf{l})^2$, $k_x = \mathbf{Q}$, $k_y = \mathbf{q}$, $\varepsilon = (m_\perp/m_\parallel) - 1 > 0$ and $\lambda^{-2} = 8\pi|\Psi|^2 e^2/mc^2$. The energy of a vortex is given as a function of its orientation by[2-4]

$$E_v^0 = \frac{\Phi_0}{(4\pi\lambda)^2}\sqrt{\cos^2\theta + (m_\parallel/m_\perp)\sin^2\theta}\,\ln\kappa(\theta), \quad (3)$$

where θ is the angle between the vortex axis $\mathbf{l}$ and the anisotropy axis $\vec{\nu}$.

In the case of an inclined vortex, we have a special plane, $(\vec{\nu}\mathbf{l})$, and the minimum energy is exhibited not by a solitary vortex but by a vortex which is part of a vortex chain lying in the $(\vec{\nu}\mathbf{l})$ plane. The energy of a vortex in such a chain, with a period a, can be written

$$E_v = E_v^0 + \frac{\Phi_0}{8\pi}\Big(\frac{1}{a}\sum_{Q=\frac{2\pi n}{a}}\int\frac{dq}{2\pi}\mathbf{lh}_{qQ} - \iint\frac{dq\,dQ}{(2\pi)^2}\mathbf{lh}_{qQ}\Big)$$

$$= E_v^0 + \frac{\Phi_0^2}{16\pi\lambda^2}\int\frac{dx}{2\pi}\Big(\frac{\coth(\frac{\tilde{a}}{2}\sqrt{x^2+1}) - 1}{\sqrt{x^2+1}} - \frac{x^2}{x^2 - (\nu_z/\nu_x)^2}$$

$$\times\Big[\frac{\coth(\frac{\tilde{a}}{2}\sqrt{x^2+1}) - 1}{\sqrt{x^2+1}} - \frac{\coth(\frac{\tilde{a}}{2}b\sqrt{(1+\epsilon)x^2+1}) - 1}{b\sqrt{(1+\epsilon)x^2+1}}\Big]\Big), \quad (4)$$

where $\tilde{a} = a/\lambda$ and $b = (1 + \epsilon\nu_z^2)^{-1/2}$. As can be seen from (4), in the case $\tilde{a} \gg 1$ the component representing the vortex interaction energy is negative, and the minimum energy of a vortex corresponds to a finite $\tilde{a}$. This result agrees with the conclusion reached in Ref. 5: that the magnetic field of an inclined vortex reverses at large distances.

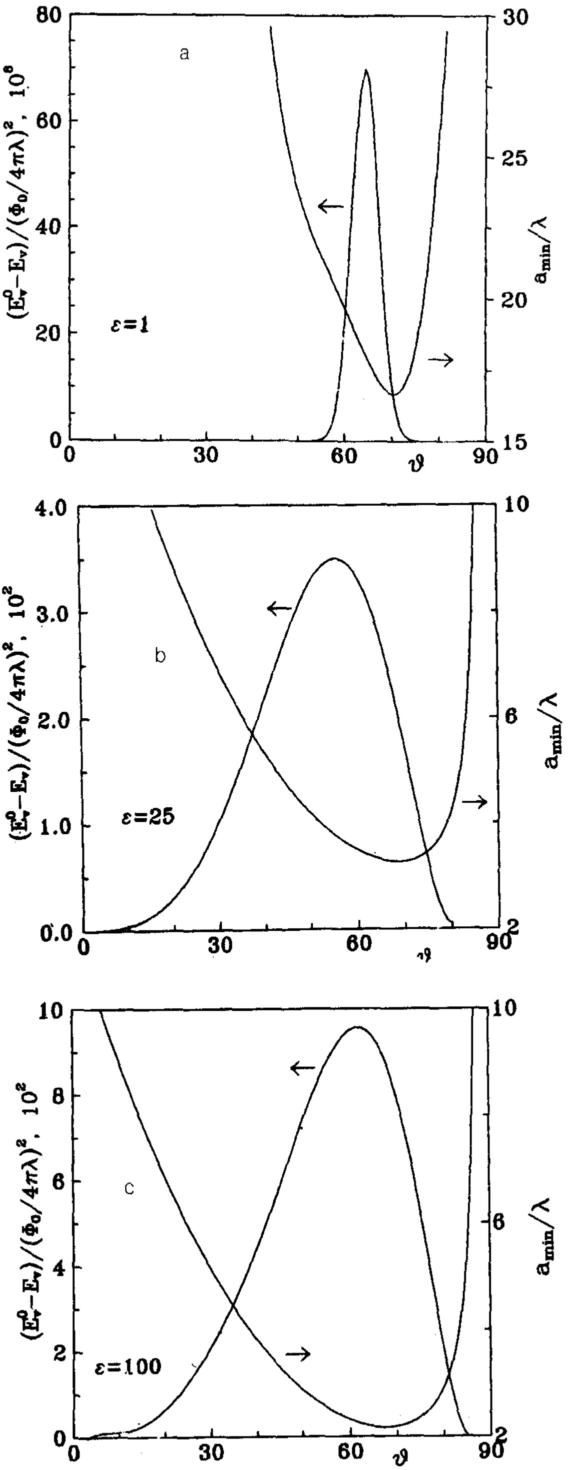
a
$(E_v^0 - E_v)/(\Phi_0/4\pi\lambda)^2,\ 10^8$
a_{min}/λ
$\varepsilon=1$
ϑ
0
20
40
60
80
30
60
90
15
20
25
30
b
$(E_v^0 - E_v)/(\Phi_0/4\pi\lambda)^2,\ 10^2$
a_{min}/λ
$\varepsilon=25$
ϑ
0.0
1.0
2.0
3.0
4.0
2
6
10
c
$(E_v^0 - E_v)/(\Phi_0/4\pi\lambda)^2,\ 10^2$
a_{min}/λ
$\varepsilon=100$
ϑ
2
4
6
8
10

FIG. 1

Since the minimum energy is that of a vortex which is part of a chain, the lower critical field H_{c1} should correspond not to a solitary vortex but to a vortex chain. In other words, the situation favors the appearance not of a single vortex but of a vortex chain. The problem of the minimum energy of a vortex which is part of such a lattice can be solved quite easily through a numerical minimization of expression (4) with respect to $\tilde{a}$. As a result, we also find the equilibrium period $\tilde{a}_{\min}$ of the vortex chain. The results are shown in Fig. 1.

It can be seen from these results that the energy of a vortex decreases to the greatest extent at angles θ near 60°, but this decrease is comparatively small: Expression (3) for E_v^0 contains a large logarithmic factor $\ln \varkappa(\theta)$, which is not present in the difference $E_v(\theta) - E_v^0(\theta)$.

Nevertheless, this circumstance tells us that the field H_{c1} does decrease slightly and does correspond to the penetration of specifically a chain of vortices into the sample.

A system of parallel chains arises in fields slightly above H_{c1}: In the first approximation, the vortex period $a_{\min}$ does not change, while the distance between chains, L, is determined by the force of their mutual repulsion.

We can find the energy component which comes from the repulsion of chains by evaluating the energy of a single vortex in a system of chains which lie at distances $L \gg a_{\min}$:

$$E_v = \frac{\Phi_0}{8\pi a L} \sum_{Q=\frac{2\pi n}{a}} \sum_{q=\frac{2\pi k}{L}} \mathrm{h}_{qQ}. \tag{5}$$

First carrying out the summation over q in (5), we find, using the Poisson formula,

$$\tilde{E}_v = E_v + \frac{\Phi_0^2}{8\pi a\lambda} \frac{\nu_x^2}{\sqrt{1+\epsilon}} \exp(-\tilde{L}/\sqrt{1+\epsilon}), \tag{6}$$

where $\tilde{L} = L/\lambda$. The last term in (6) gives us the increase in the energy of the vortex due to the interaction of chains in the asymptotic limit $L \gg a^2/\lambda$, λ.

Using (6), one easily finds[6] that the magnetic induction B in fields just slightly above H_{c1} is

$$B = \frac{\Phi_0}{a\lambda\sqrt{1+\epsilon}} \ln^{-1}\left[\frac{\Phi_0 \nu_x^2}{2a\lambda(1+\epsilon)} (H - H_{c1})^{-1}\right]. \tag{7}$$

For isotropic superconductors we would have[6] $B \sim \ln^{-2}[(H - H_{c1})^{-1}]$.

In layered high-T_c superconductors of the $(\mathrm{Re})\mathrm{Ba_2Cu_3O_7}$ type, the anisotropy parameter is[7] $\varepsilon \approx 25$, and in this case [Fig. 1(b)] we would expect a significant decrease in the energy of a vortex in the chain at an angle $\theta \approx 60°$.

For a sample which is an ellipsoid of revolution with axis coinciding with the anisotropy axis $\vec{\nu}$ and with a demagnetizing factor n along this axis, the orientation angles of the vortex, θ, and of the external field, φ, at $H = H_{c1}$ are related by

$\tan\theta = 2(1-n/1+n)(1+\varepsilon)\tan\varphi$, and the magnitude of the lower critical field is

$$H_{c1}(\varphi) = \frac{\Phi_0 \ln(\kappa(\theta))(1-n^2)}{4\pi\lambda^2\sqrt{4(1-n)^2(1+\epsilon)\sin^2\varphi + (1+n)^2\cos^2\varphi}}.$$

As a result, at angles φ corresponding to $\theta = 60°$ in a field $H = H_{c1}$ we would expect chains of vortices with a period $a \approx 2\lambda$ to appear far from each other (for $YBa_2Cu_3O_7$ we would have $\lambda \sim 300$–500 Å; Ref. 8).

Chains of this sort might be detected by magnetooptic methods or decoration methods. It would also be interesting to study an inclined vortex lattice in fields close to H_{c1} by neutron diffraction.

The penetration at $H = H_{c1}$ of vortex chains, rather than individual vortices, might also be manifested in magnetomechanical experiments.

The effects discussed above are not unique to layered superconductors. They might also be manifested in any anisotropic uniaxial superconductor with $m_{\parallel} < m_{\perp}$ (in the case $m_{\parallel} > m_{\perp}$, the vortices in the chains would repel each other).

We wish to thank A. A. Abrikosov for useful discussions.

[1]L. N. Bulaevskiĭ, Usp. Fiz. Nauk **116**, 449 (1975) [Sov. Phys. Usp. **18**, 514 (1975)].
[2]V. G. Kogan, Phys. Rev. B **324**, 1572 (1981).
[3]A. M. Grishin, Fiz. Nizk. Temp. **9**, 277 (1983) [Sov. J. Low Temp. Phys. **9**, 138 (1983)].
[4]A. V. Balatskiĭ *et al.*, Zh. Eksp. Teor. Fiz. **90**, 1478 (1986) [Sov. Phys. JETP **63**, 866 (1986)].
[5]A. M. Grishin *et al.*, in *Proceedings of the Second All-Union Conference on the Physics of High T_c Superconductors*, Kiev, 1989, p. 62.
[6]A. A. Abrikosov, *Fundamentals of the Theory of Metals*, Nauka, Moscow, 1987.
[7]D. E. Farrell *et al.*, Phys. Rev. Lett. **61**, 2805 (1988).
[8]L. P. Gor'kov and N. B. Kopnin, Usp. Fiz. Nauk **156**, 117 (1988) [Sov. Phys. Usp. **31**, 850 (1988)].

Translated by Dave Parsons