Effect of thermal fluctuations on pinning of 2D vortex lattice

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The temperature dependence of the critical current is derived for a 2D superconductor in a magnetic field. There are two characteristic temperatures at which there are substantial changes in this behavior. In the region in which fluctuations have a strong effect, there is a field interval in which the critical current increases with increasing field.

Because of the high transition temperature and short coherence length of the oxide superconductors, thermal fluctuations influence the superconducting properties of these materials. One of the most obvious fluctuation effects is the appearance of a strong temperature dependence of the critical current in a magnetic field at temperatures well below the superconducting transition temperature.[1-3] A suppression by fluctuations of the force pinning Abrikosov vortices in a 3D superconductor was studied first by Vinokur and Feigel'man.[4]

In high-T_c superconducting films with a thickness smaller than the vortex lattice parameter, the fluctuations of the lattice are two-dimensional, and one would expect them to have an even stronger effect on the pinning force. In addition, layered high-T_c superconductors with a very weak Josephson interaction between layers have now been reported. If the inhomogeneities of these superconductors are sufficiently pronounced, a pinning of the vortex lattice occurs in each layer, in a manner independent of the other layers. It is thus worthwhile to determine the temperature dependence of the critical current for the mixed state of a 2D superconductor.

The elastic energy of the vortex lattice in an inhomogeneous 2D superconductor is

$$E_{el}=\int d\mathbf{r}\{\frac{C_{66}}{2}(\partial_i u_j)^2+\frac{C_{11}}{2}(\partial_i u_i)^2+v(\mathbf{r})p(\mathbf{r}-\mathbf{u})-d(\mathbf{B}\times\mathbf{j})\mathbf{u}/c\}, \quad (1)$$

where $\mathbf{u}$ is the lattice displacement, j is the external current, and d is the distance between the superconducting layers (in the case of a layered superconductor) or the film thickness (in the case of a thin film). At fields $H_{c1}\ll B\ll H_{c2}$, the shear modulus is given by $(C_{66}=d\Phi_0 B/(8\pi\lambda)^2$, where λ is the London penetration depth. The compression modulus C_{11} is much larger than C_{66} and thus has only a slight effect on the pinning force. The function $v(\mathbf{r})$ describes random variations in the superconducting condensation energy and is characterized by the correlation function

$$\langle v(\mathbf{r})v(\mathbf{r}')\rangle=\gamma\delta(\mathbf{r}-\mathbf{r}'). \quad (2)$$

The dimensionless periodic function $p(\mathbf{r})$ describes a modulation of the amplitude of the order parameter and can be written

$$p(\mathbf{r})=\sum_{\mathbf{R}_l} f(|\mathbf{r}-\mathbf{R}_l|/\xi).$$

The summation is over the sites $\mathbf{R}_l$ of an ideal triangular lattice, ξ is the coherence length, $f(0)=0$, and $f(x)\to 1$ as $x\to\infty$.

Thermal fluctuations weaken the random potential. At a nonzero temperature, the random potential $V_{rp}(\mathbf{r},\mathbf{u})$, renormalized by the fluctuations, is $V_{rp}(\mathbf{r},\mathbf{u}) = v(r)\langle p(r-u_f-u)\rangle_t$, where $\mathbf{u}_f$ is the fluctuation displacement of the lattice, and $\langle\ldots\rangle_t$ means a thermodynamic average.

An important characteristic of pinning is the correlation radius[5] R_c, which is defined as the distance over which the lattice displacement due to inhomogeneities becomes comparable in magnitude to the range of the random potential, r_p. The correlation radius can be estimated from the condition that the elastic energy is balanced by the interaction energy with a random potential:

$$R_c\sim\frac{C_{66}r_p^2}{E_{rp}}, \quad (3)$$

where $E_{rp}S^{1/2}$ is a characteristic fluctuation of the random potential over an area S. Expanding the function $p(\mathbf{r})$ in a Fourier series, $p(\mathbf{r}) = \Sigma_{\mathbf{k}} p_{\mathbf{k}} \exp(i\mathbf{K}\mathbf{r})$ ($\mathbf{K}$ is a reciprocal lattice vector), we find the following expression for E_{rp}:

$$E_{rp}^2 = \gamma \sum_{\mathbf{K}} |p_{\mathbf{K}}|^2 \exp\left(-\frac{K^2}{2}\langle u_f^2\rangle\right). \tag{4}$$

In the homogeneous case, the mean square fluctuation $\langle u_f^2\rangle$ diverges logarithmically because of the long-wavelength soft excitations. The random potential disrupts the translational symmetry of the lattice, with the result that the divergence is cut off at a wave vector $k_c \sim 1/R_c$. The quantity $\langle u_f^2\rangle$ is thus finite and is given by

$$\langle u_f^2\rangle = \frac{T}{4\pi C_{66}} \ln(n_v R_c^2), \tag{5}$$

where $n_v = B/\Phi_0$ is the density of vortices. The range of the random potential, r_p, depends on the relations among the length scales ξ, $(\langle u_f^2\rangle)^{1/2}$, and the parameter of the vortex lattice, a:

$$r_p \approx \begin{cases} \xi, & \text{for } (\langle u_f^2\rangle)^{1/2} < \xi \\ (\langle u_f^2\rangle)^{1/2}, & \text{for } \xi < (\langle u_f^2\rangle)^{1/2} < a\,. \\ a, & \text{for } (\langle u_f^2\rangle)^{1/2} > a\,. \end{cases} \tag{6}$$

Equations (3)–(6) constitute a self-consistent system which determines the parameters R_c, $(\langle u_f^2\rangle)^{1/2}$, r_p, and E_{rp}. The critical current density j_c is proportional to the typical pinning force acting on the correlation area $(E_{rp}/r_p R_c)$:

$$j_c \approx \frac{cE_{rp}}{dBR_c r_p}\,. \tag{7}$$

We consider the field range in which the pinning of the lattice is collective at $T=0$. The dimensionless parameter $C_{66}\xi/\gamma^{1/2}$ is thus assumed to be large. The results of a solution of system (3)–(7) for various temperature ranges can be summarized as follows.

1. At $T < T_p \approx 2\pi C_{66}\xi^2/\ln(C_{66}\xi/\gamma^{1/2})$, the fluctuations can be ignored, and we find the Larkin–Ovchinnikov result for the critical current density[5]:

$$j_{c0} \approx \frac{c\gamma}{\Phi_0 C_{66}\xi}\,. \tag{8}$$

2. At $T_p < T < T_f \approx (3/4\pi) C_{66} a^2/\ln(C_{66}/(\gamma n_v^3 \xi^4))$ the summation in (4) is cut off at $K \sim 1/\langle u_f^2\rangle^{1/2}$, and a self-consistent solution of system (3)–(7) yields

$$j_c = j_{c0}\left\{\frac{T}{T_p}\left(1 + \frac{3\ln(T/T_p)}{2\ln(C_{66}\xi/\gamma^{1/2})}\right)\right\}^{-5/2} \tag{9}$$

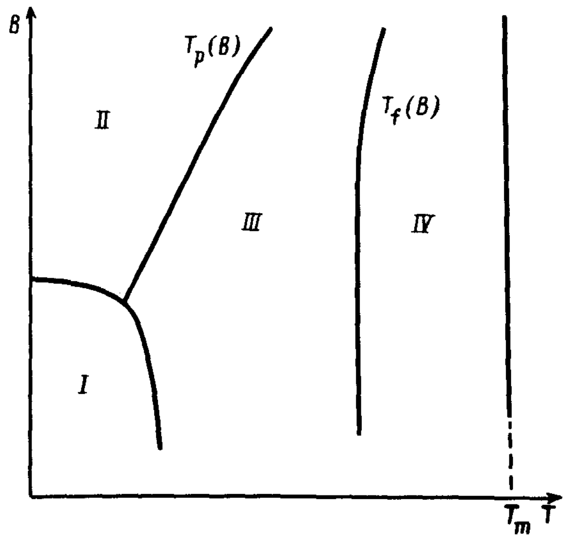

FIG. 1. Regions differing in the nature of the pinning of the vortex lattice. I—Individual pinning; II, III, IV—collective pinning; II—weak fluctuation effect; III—thermal depinning; IV—fluctuation region.

3. At $T > T_f$, we need retain in the sum in (4) only the six terms which correspond to the minimum value of the reciprocal lattice vector $K_0 = 4\pi/(\sqrt{3a})$, and the solution of the system becomes

$$j_e = j_{c0} \{n_v \xi^2\}^{5/2} \{\frac{C_{66}^2}{\gamma n_v^3 \xi^4}\}^{-\frac{\Delta}{1-\Delta}}, \tag{10}$$

where $\Delta = 2\pi T/(3C_{66}a^2)$. A behavior of the type in (10) is typical of the fluctuation region of 2D systems in which a continuous degeneracy has been lifted by a weak perturbation.[6] At the melting point of the lattice,[7] $T = T_m = C_{66}a^2/(4\pi)$ the exponent $\Delta/(1-\Delta)$ has a universal value of 1/5. Free dislocations appear at this point, causing the critical current to vanish. The characteristic temperatures (T_p and T_f) are determfined primarily by the properties of the material and the magnetic field; they depend only weakly (logarithmically) on the degree of inhomogeneity, characterized by the parameter γ. In the region in which fluctuations have a strong effect, the critical current increases with increasing magnetic field in accordance with $j_c \sim B^{3/2}$. This increase is a consequence of a stiffening of the lattice, which weakens the fluctuations and increases the amplitude of the random potential. Figure 1 shows the regions in the temperature-(magnetic field) plane in which the critical current behaves in substantially different ways.

We conclude with an estimate of the characteristic temperatures. For high-T_c superconducting films of thickness $d \sim 100$ Å the depinning temperature T_p increases linearly with the field, with a slope $dT_p/dB \gtrsim 5\ K/T$. The boundary of the fluctuation region, T_f, depends weakly on the field and has a value of about 50 K.

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