Electromagnetic properties of ceramic high-T_c superconductor in critical state

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A method is proposed for directly measuring the critical current density j_c of a ceramic high-T_c superconductor as a function of the magnetic field H through the use of a low-frequency electromagnetic signal as a probe. In a model of the critical state, the surface impedance of a ceramic sample is proportional to the function $j_c^{-1}(H)$.

1. According to the present understanding, a ceramic high-T_c superconductor can be thought of as a system of superconducting grains connected by weak (Josephson) links. To a large extent, the electromagnetic properties of such a system are determined by the relation between the parameters a and λ_j, where a is a characteristic

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size of the structural cell, $\lambda_j \approx (\hbar c^2 a/8\pi e I_c)^{1/2}$ is the Josephson penetration depth, I_c is the critical current of the weak link, e is the elementary charge, and c is the velocity of light. Under the inequality

$$a \ll \lambda_j, \tag{1}$$

a magnetic flux quantum penetrating into the sample spans a large number, $\approx(\lambda_j/a)^2$, of contacts. In terms of macroscopic properties, such a medium would thus be similar to an inhomogeneous type-II superconductor. In the opposite case,

$$\lambda_j \ll a, \tag{2}$$

the magnetic flux would "break through" the weak links in succession as it penetrated into the sample.

Despite the fundamentally different physics in situations (1) and (2), the electrodynamic equations are formally identical. As Dersch and Blatter[1] have shown, in both cases (1) and (2) the electrodynamic properties of a ceramic high-T_c superconductor can be described well by Bean's model of the critical state. In that model, the electromagnetic field distribution is described by the equation

$$c \text{ curl } \mathbf{B} = 4\pi\mu j_c(\mathbf{E}/E). \tag{3}$$

Here $\mathbf{B}$ is the magnetic induction, $\mathbf{E}$ is the electric field, μ is the magnetic permeability of the ceramic under the assumption that there are no Josephson links, and j_c is the critical current density. An important circumstance is that the critical current is extremely sensitive to the magnetic field:

$$j_c = j_c(B). \tag{4}$$

It is in the form of constitutive equation (4) that we see the physical difference between cases (1) and (2). Determining this functional dependence is thus important for describing a ceramic. The practical side of this problem is also important, since (4) is pertinent to the critical transport currents.

In previous experiments, an average value of j_c along the cross section of the sample has been determined as a function of the external magnetic field, usually by the four-terminal method or the inductance method. In Ref. 2, the function $j_c(B)$ was found from measurements of the dynamic susceptibility of a sample during the flow of a transport current. In the present letter we suggest a contactless method for finding $j_c(B)$ directly, by probing the sample with a weak electromagnetic signal in a static external magnetic field.

2. We consider a cylindrical sample of radius R in a magnetic field

$$H(t) = H + h\cos\omega t,$$

directed along the axis of the cylinder (the z axis). The electromagnetic field in the sample is axially symmetric, and in a cylindrical coordinate system it depends on the coordinate r alone. The magnetic field has only a z component in this case, and the electric field only an azimuthal component. From (3) we can easily derive equations for the variable component of the magnetic induction B:

$$c\,\partial b(r,t)/\partial r = \pm 4\pi\mu j_c(B+b), \tag{5}$$

where $B = \mu H$. The boundary condition on (5) is

$$b(R,t) = \mu h \cos\omega t. \tag{6}$$

We assume that the field amplitude h is so low that we can ignore the b dependence of j_c in (5). As a rule, this condition will hold if h is below 1 Oe.

Omitting the simple calculations, we write the result for the response of the sample to an external field $H(t)$. The electric field at the surface of the sample, which is related to the alternating magnetic flux Φ by

$$E(t) = -(\partial\Phi/\partial t)/(2\pi R c), \tag{7}$$

is given by

$$E(t) = \frac{\mu\omega h^2}{8\pi j_c(B)}(1-\cos\omega t)\sin\omega t, \quad 0<\omega t<\pi$$
$$E(t+\pi/\omega) = -E(t). \tag{8}$$

Terms on the order of $hc/16\pi j_c R$ have been omitted from (8); in samples with $R \geqslant 1$ mm, with $h \leqslant 1$ Oe and $j_c \geqslant \mathrm{A/cm^2}$, such terms are definitely small.

By measuring the electric field at its maximum,

$$E_{max} = \frac{3^{3/2}}{32\pi}\,\frac{\mu\omega h^2}{j_c(B)}, \quad B = \mu H, \tag{9}$$

we can thus find the dependence $j_c(B)$. A numerical estimate based on (9) shows that the voltage induced in a 100-turn measurement coil with $\omega/2\pi = 1$ kHz, $h = 1$ Oe, $\mu = 0.5$, $R = 0.5$ cm, and $j_c = 200$ A/cm^2 would be about 20 μV.

It is of course not necessary to study specifically $E(t)$; it would be sufficient to measure any harmonic of the electric field. For example, the surface impedance determined by the first harmonic of $E(t)$ is related to the critical current by

$$Z = \frac{2}{3\pi}\,\frac{\mu\omega h}{c j_c(B)}(1 - 3\pi i/4). \tag{10}$$

Even at small amplitudes h, the impedance would be a nonlinear function of h, because the current in (5), caused by the alternating electromagnetic field, does not depend on the amplitude of this field. The phase factor of impedance (10) is also unusual.

This method for measuring $j_c(B)$ on the basis of (9) and (10) would be simple to implement experimentally. It would not require fabricating electrical contacts, and well-developed methods for measuring the field $E(t)$ and the impedance are already available.

3. In conclusion we would like to call attention to some opportunities which are

opened up by this new method for studying the electrodynamic properties of ceramic high-T_c superconductors. It follows from the present letter that, even in the very simple case of extremely small amplitudes of the exciting field, the field $E(t)$ would carry important information about the critical state of the sample. It would clearly be interesting to carry out a more detailed study of the response of a sample to an electromagnetic perturbation in very nonlinear regimes, during the flow of a transport current, etc.

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[1] H. Dersch and G. Blatter, Phys. Rev. B **38**, 11391 (1988).
[2] L. M. Fisher *et al.*, Adv. Cry. Eng.-Materials **36**, 173 (1989).

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