Scattering of neutrinos by nuclei in matter

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The elastic scattering of neutrinos by nuclei in a nonideal medium at a small momentum transfer is reduced to the emission and absorption of phonons. The cross section for νA scattering in this case is considerably smaller than that for νA scattering in an ideal gas.

In problems involving the diffusion of neutrinos in dense collapsing matter in the stage of neutrino opacity,[1,2] the coupling constant is much greater than unity ($\Gamma = Z^2 e^2 n_0^{1/3}/T \gg 1$; n_0 is the equilibrium density of nuclei, e is the charge of an electron, and T is the temperature of the medium). The momentum transfer in the scattering of neutrinos with an energy $E \lesssim 100$ MeV satisfies $k \ll Mu$, where M is the mass of a nucleus, and u is the sound velocity in the medium. The interaction between the nuclei in matter causes the spectrum of elementry excitations of the medium to become sonic at low momenta, and the scattering of neutrinos should reduce to the emission and absorption of phonons.[1)]

The expression for the cross section for scattering by 2 single nucleus,[3]

$$\sigma_{\nu A} = \frac{1}{4\pi}[Z(1-4\xi) - N]^2 G_F^2 E^2 \qquad (1)$$

($G_F = 10^{-5} m_p^{-2}, m_p$ is the mass of the proton, $\xi = \sin^2\theta_W \approx 0.22$, θ_W is the Weinberg angle, N is the number of neutrons in the nucleus, $z = A = N$ is the atomic number, and E is the energy of the incident neutrino; we are using a system of units with

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$\hbar = c = 1$), which is valid only for an ideal gas of spin-zero nuclei, cannot be used in calculations on the neutrino transparency of the central part of a collapsing star. At a small momentum transfer, a neutrino is scattered not by individual nuclei but by perturbations of the density of nuclei.

Taking into account the coherent interaction of neutrinos with all the nucleons of a nucleus through neutral currents, we can write the following expression for the interaction of neutrinos with fluctuations of the density of nuclei[2),3)]:

$$\hat{\mathcal{H}} = \frac{1}{2\sqrt{2}} G_F [Z(1-4\xi) - N](\bar{\Psi}_\nu \gamma_0 (1-\gamma_5) \Psi_\nu) \delta n(\mathbf{r}, t). \quad (2)$$

The nuclei of the medium are assumed to be nonrelativistic, so their 4-current has only a temporal component $J_0 = (\Psi^+ \Psi)$, which is, after an averaging over the medium, equal to the density of nuclei.

The square of the matrix element for a transition of a neutrino from the state $p = (E, \mathbf{p})$ to the state $p' = (E' \mathbf{p} - \mathbf{k})$, where $E = |\mathbf{p}|$ and $E' = |\mathbf{p} - \mathbf{k}|$ (we are ignoring the mass of the neutrino), averaged over the fluctuations of the density of the medium, is

$$\overline{|M_{pp'}|^2} = [Z(1-4\xi) - N]^2 G_F^2 \{2EE' + \frac{1}{2}[(E-E')^2 - \mathbf{k}^2]\} \langle \delta n^2 \rangle_{E-E', \mathbf{k}} . \quad (3)$$

where $\langle \delta n^2 \rangle_{\omega, \mathbf{k}}$ is the spectral distribution of fluctuations of the density of particles. In the case $k \ll Mu$, the correlation function of the density fluctuations has sharp peaks at frequencies corresponding to acoustic vibrations[5] $\omega = ku$:

$$\langle \delta n^2 \rangle_{E-E', \mathbf{k}} = \frac{\pi n_0 k}{Mu} \{(N_k + 1) \delta (E - E' - ku) + N_k \delta (E - E' + ku)\}, \quad (4)$$

where $N_k = [\exp(ks/T) - 1]^{-1}$ is the number of phonons of frequency ku at thermodynamic equilibrium.

Ignoring the energy of the sound wave in comparison with the energy of the neutrino ($ku \ll E$), we can assume the scattering to be elastic, i.e., we can replace $\delta(E - E' \mp ku)$ by $\delta(E - E')$; using (3) and (4), we can calculate the differential cross section per nucleus for the scattering of neutrinos through an angle θ:

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\sqrt{2}}{8\pi} [Z(1-4\xi) - N]^2 G_F^2 \frac{E^3}{Mu} [f(\theta) + \frac{1}{2}](1 + \cos\theta)(1 - \cos\theta)^{1/2}, \quad (5)$$

where

$$f(\theta) = \{\exp[\frac{Eu}{T}\sqrt{2(1-\cos\theta)}] - 1\}^{-1}. \quad (6)$$

At $T = 0$ we would have $f(\theta) = 0$, and the scattering of a neutrino by nuclei would reduce to the spontaneous emission of phonons. In this case the total cross section for elastic scattering by a single nucleus is

$$\sigma = \frac{2}{15\pi}[Z(1-4\xi)-N]^2 G_F^2 \frac{E^3}{Mu}. \tag{7}$$

At $T \gg Eu$, stimulated emission and stimulated absorption of phonons are predominant. The neutrino scattering cross section per nucleus in this case is

$$\sigma = \frac{1}{4\pi}[Z(1-4\xi)-N]^2 \frac{T}{Mu^2} G_F^2 E^2. \tag{8}$$

A comparison of (8) with (1) shows that only if the medium has a very high temperature $T \sim Mu^2$ (for collapsing matter, $Mu^2 \sim 10$ MeV) is the cross section for scattering of neutrinos by nuclei in matter comparable to the cross section for scattering by an isolated nucleus. At $T \ll Mu^2$, the cross section for the scattering of neutrinos by nuclei in matter is much smaller than (1). This point should be kept in mind in calculations of the neutrino transparency of a collapsing star.

[1)] There is an analogy here with the scattering of slow neutrons in matter. If a neutrino is scattered by a limited volume of matter, e.g., by a crystal of finite dimensions, there may also be a coherent scattering by a crystal lattice, with a transfer of momentum to the entire crystal. If, on the other hand, the neutrino is propagating through an unbounded crystal, there will be no elastic scatering by an ideal crystal lattice (by analogy with Bloch electrons in a periodic field).

[2)] We are using the standard representation of Dirac γ matrices, with $\gamma_5 = \gamma_5^+ = i\gamma_0\gamma_1\gamma_2\gamma_3$.

[3)] At distances greater than the electron Debye length R_D, the weak interaction of a neutrino with a nucleus is screened by the electrons of the medium.[4] This screening is important only at neutrino energies $E \sim R_D^{-1}(R_D^{-1} \lesssim 3\text{–}5$ MeV) for a collapsar, and we are ignoring it here.

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