Refinement of QCD parton model

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Corrections to the naive parton model for virtuality and for the direction of the polarization vector of a soft gluon are discussed.

1. We know that in quantum chromodynamics (QCD) the cross sections for processes for which small distances are important (e.g., the production of a pair of heavy quarks or jets of hadrons with large transverse momenta $p_t \gg m$) can be described in the customary parton form[1-3]:

$$E_i d\sigma/d^3p_i = \int dx_1 dx_2 D_a(x_1, Q^2) D_b'(x_2, Q^2) E_i d\hat{\sigma}/d^3p_i, \tag{1}$$

where $d\hat{\sigma}$ is the cross section for the hard parton–parton interaction (involving the emission of, for example, a large-p_{ti} jet or a pair of heavy quarks), and the functions $D_a(x_1,Q^2)$ and $D_b(x_2,Q^2)$ are the probabilities for finding suitable partons with momentum fractions x_1 and x_2 in the wave functions of the initial hadrons, a and b [Fig. 1(a)].

Expression (1) holds in the leading-log approximation of QCD, in which the virtualities of the colliding partons (gluons or quarks) satisfy $q_1^2, q_2^2 \ll Q^2$, where the right side is the characteristic momentum transfer in the matrix element of subprocess $d\hat{\sigma}$ (for the production of a pair of heavy quarks $Q\tilde{Q}$ we would have $Q^2 \sim m_Q^2$, while for the emission of a jet of hadrons with large p_t we would have $Q^2 \sim p_t^2$). At high energies, however, at which the hard interaction $d\hat{\sigma}$ corresponds to only a very small fraction of the initial energy ($x_1, x_2 \ll 1$), the important virtualities q_1^2 and q_2^2 are not very small, and corrections should be made in parton formula (1) to allow for the fact that the momenta of the colliding partons (the gluons q_1 and q_2) do not lie on the mass shell and the fact that the cross section $d\hat{\sigma}$ differs from that for the collision of massless unpolarized gluons.

In addition, it would be useful to recall the polarization of gluons q_1 and q_2. In terms of its derivation, expression (1) is completely equivalent to the equivalent-photon method which is widely used in quantum electrodynamics. The polarization vector of the equivalent photon, $\mathbf{e}_i$, however, lies in the reaction plane (in which it was emitted; $\mathbf{e}_i \| \mathbf{q}_{it}$), and this alignment affects the angular distributions of the particles formed in subprocess $d\hat{\sigma}$. In the case of equivalent photons, this effect has been taken into account for a long time now.[4,5] Let us demonstrate how it may be manifested in QCD.

2. As a very simple example we consider the photoproduction of a pair of heavy quarks $Q\tilde{Q}$ [Fig. 1(b)]. Participating in the hard interaction here are an unpolarized initial photon γ and a gluon q. The cross section for the production of quark–antiquark pair Q_1Q_2 satisfies the proportionality

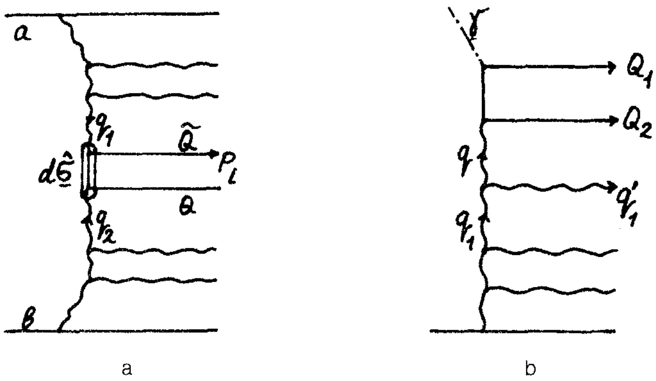

FIG. 1. Production of a pair of heavy quarks, $Q\tilde{Q}$, (a) in a hadron–hadron interaction and (b) in a photoproduction process.

$$\frac{d\hat{\sigma}}{dz dQ_t^2} \propto \frac{z^2 + (1-z)^2}{m_t^2 m_q^2} - \frac{2z(1-z)m^2}{q_t^2}\left(\frac{1}{m_t^2} - \frac{1}{m_q^2}\right)^2 = I \tag{2}$$

[z is the fraction of the momentum of he γ ray carried off by quark Q_2; $Q_t = Q_{1t}$ is the transverse momentum of the antiquark; $m_t^2 = m^2 + Q_t^2$;$m_q^2 = m^2 + (Q-q)_t^2$ and $m = m_Q$ is the mass of the quark].

For very small virtualities, $q^2 \ll m^2$, proportionality (2) takes the form

$$I = [z^2 + (1-z)^2 + 8z(1-z)m^2 Q_t^2 \cos^2\varphi / m_t^4]/m_t^4 , \tag{3}$$

(where φ is the angle between the vectors $\mathbf{q}_t$ and $\mathbf{Q}_t$). If we had averaged the cross section $d\hat{\sigma}$ over the polarizations of the "equivalent" gluon q, we would have found

$$\langle I \rangle = [z^2 + (1-z)^2 + 4z(1-z)m^2 Q_t^2 / m_t^4]/m_t^4 . \tag{4}$$

For the typical values $Q_t = m$ and $z = 1/2$ the ratio of cross sections described by expressions (3) and (4) is

$$I/\langle I \rangle = 2(1 + \cos^2\varphi)/3. \tag{5}$$

We see that the actual distribution of quarks in the azimuthal plane is not isotropic, and the probability for the emission of these quarks along the axis parallel to $\mathbf{q}_t$ is substantially larger [by a factor of two in case (5)] than in the perpendicular direction. This is a 100% effect in comparison with the ordinary parton model. We can avoid dealing with this effect by taking an average over the azimuthal angle of the vector q_t and calculating only the total photoproduction cross section, but this effect is present in inclusive spectra. Even for parametrically small $q_t \ll m$, the direction of q_t is actually always known, since the transverse momentum q_t is equal to the sum of the momenta of the quarks: $(\mathbf{Q}_1 + \mathbf{Q}_2)_t = \mathbf{q}_t$.

3. How does the cross section for the hard interaction, $d\hat{\sigma}$, depend on the virtuality of the gluon, q^2? For clarity, we consider the total cross section for the production of the $Q\bar{Q}$ pair, integrated over the momentum of the quark, d^3Q_1:

$$\hat{\sigma} \propto J = \int I dz d^2 Q_{1t}/\pi = \frac{2}{3q^2}\left[\frac{1-1/y}{\sqrt{1+4/y}} \ln \frac{(1+y)\sqrt{1+4/y}+y+3}{\sqrt{1+4/y}-1} + 1\right]. \quad (6)$$

If the ratio $y = |q^2|/m^2$ is very small, we have $J = (7/9)m^2$, and the cross section in (6) falls off in proportion to $1/y$ with increasing $|q^2|$. In the important range of $q^2 (y < 15)$, we can quite accurately ($\lesssim 0.02$) approximate expression (6) for J by

$$J = \frac{7/9m^2}{1+y/6.7}, \quad (7)$$

From this result we see that the logarithmic integration over the momentum q^2, which is contained in the structure function

$$xD(x, Q^2) = \int^{Q^2} \varphi(x, Q^2) dq^2/q^2, \quad (8)$$

continues to extremely large values $|q^2| \sim 6m^2$, and only at $|q^2| > (6\text{–}7)m^2$ does the integral $D(x,Q^2)\hat{\sigma} \propto \int \varphi(x,q^2) J(y = |q^2|/m^2) dq^2/q^2$ begin to converge, because of the decrease in the cross section $\hat{\sigma} \propto J \propto 1/q^2$. As usual in the leading-log approximation, we are assuming here that the function $\varphi(x,q^2)$ depends on q^2 rather weakly, merely logarithmically, everywhere except at small values $q^2 \sim \mu^2$, where μ is the lower limit of the logarithmic integration of dq^2/q^2.

Having written out the explicit functional dependence of the cross section for the hard interaction, $\hat{\sigma}$, on the virtuality q^2, we are thus in a position to estimate an upper limit on the integration of dq^2/q^2, i.e., the argument of the structure function $D(x,Q^2)$: $Q^2 \sim 6.7\ m^2$. As a rule, one chooses $Q^2 = (I-4)m^2$ in the leading-log approximation. A refinement of this sort may prove important at small values of x (i.e., at high energies), since the effective infrared cutoff of the integral of dq^2/q^2 occurs at momenta $q^2 \approx q_0^2(x) \gg \mu^2$ because of absorption corrections (whose role increases sharply as q^2 decreases).[6] In terms of the function φ, this assertion can be formulated by approximating $\varphi(x,q^2)$ by the expression

$$\varphi(x, q^2) = q^2/(q_0^2(x) + q^2). \quad (9)$$

The value of $q_0(x)$ calculated in the leading-log approximation increases rapidly with decreasing momentum fraction x, $\ln q_0^2(x) = 3.56\sqrt{\ln 1/x}$ (Ref. 6), and has the values $q_0^2(x) = (2\ \text{GeV})^2$ and $(4\ \text{GeV})^2$ for $x = 10^{-2}$ and $x = 10^{-3}$, respectively.[1)] The value $q_0^2 = 4\ \text{GeV})^2$ is large enough even for the production of $b\bar{b}$ quarks. Explicitly incorporating the virtuality dependence of the cross section $\tilde{\sigma}$, we find that the result derived in the leading-log approximation, $-\ln Q^2/q_0^2 = \ln 4m^2/q_0^2$, is increased by more than 20%. For the case of the photoproduction of charm ($x = 10^{-2}$, in particular, corresponds to the production of a $c\bar{c}$ pair by a photon with an energy ≈ 1 TeV in the rest frame of the target proton), the effect is even greater. The cross section increases by a factor of 1.6, since in this case we are left with almost no region of a logarithmic integration of dq^2/q^2.

4. These examples show that the refinement of the naive parton model through the incorporation of virtuality and the polarization density matrix of the colliding partons will cause a significant change in the final result in many cases. To a large extent, the effects discussed above are taken into account in calculations of corrections of higher order in the constant α_s (Refs. 8 and 9). In the next order in α_s, such subprocesses as the production of three particles are also taken into consideration in the problem. For example, there is the production of a gluon q'_1 and of quarks Q_1,Q_2 in the collision of a gluon q_1 with a photon [Fig. 1(b)]. Since the matrix element for the subprocess $q_1\gamma\to Q_1\tilde{Q}_2 g'_1$ can be calculated exactly (in the given order in α_s), the polarization and virtuality of the internal gluon, q, are taken into account correctly in this case. In the preceding step, however, we are left with the question of incorporating the polarization and virtuality of the gluon q_1. The refinement of the parton model proposed in the present paper is thus advantageous in this case also.

This entire discussion also applies to the production of heavy quarks in hadron–hadron interactions [Fig. 1(a)] and to other hard processes. Because of the large number of possible invariants produced by the polarization vectors and transverse momenta of the two colliding partons [q_1 and q_2 in Fig. 1(a)], however, the expressions are too lengthy for this letter. We have accordingly restricted the discussion to the very simple and graphic process of photoproduction.

[1)]A phenomenological analysis of the data at the energies of the $S_{p\bar{p}}S$ collider, $\sqrt{s}=200$–900 GeV, has yielded the preasymptotic corrections for the quantity $q_0(x)$ and the expression[7] $q_0^2(x) = Q_0^2 + \Lambda \exp(3.56\sqrt{\ln 1/3x})$, where $Q_0^2 = 2$ GeV^2, and $\Lambda = 52$ MeV.

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