Second lasing threshold: onset of coherence

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A new physical effect is discussed: a second threshold for lasing, at which coherence arises in the laser light. The laser light field increases to a value corresponding to a coherent state at a pump level above a threshold. At a pump level above the second threshold, the laser light is in a state close to coherence. In the region between the two thresholds, the state of the laser light is a substantially quantum-mechanical state, and far from coherent. The position of the second threshold is found. This position is found as a function of the superposition of active atoms in the state, i.e., as a function of the admixture of the lower working state in it.

Scully and Lamb[1,2] established that the average value of the electric field decreases during lasing. Under the conditions which they considered, the result is the appearance of a macroscopic quantum-mechanical state above the threshold. This state is far from coherent, with a field expectation value close to zero, while the field energy is nonzero.[4,5] Below we show that the field increases to its classical value (as in a coherent state) at certain values of the pump level which are above a threshold.

Since the field determines the coherence properties of various types of radiation, this value of the pump level could be called a "second threshold," at which coherence arises in the laser. That the laser light is nonclassical near the threshold was pointed out in Refs. 3 and 4; the transition of the light to a classical state (a second threshold) was discussed in Ref. 5.

In the present letter we use the Scully-Lamb approach to incorporate the possibility that the interaction of the active atoms with the field is initially in a superposition state

$$|\Psi\rangle=\beta|b\rangle+\gamma|g\rangle,\ (|\beta|^2+|\gamma|^2=1,\ \ \Psi=\arg(\beta^*\gamma),\ \epsilon=|\beta^*\gamma|), \tag{1}$$

where $|b\rangle$ and $|g\rangle$ are the upper and lower working states of the active atom. The interaction of these atoms with the field is taken into account in all orders of perturbation theory in the rotating-field approximation. In this case the threshold pump level v_0 (the number of active atoms which interact with the field per unit time) is determined by the condition

$$v_0(|\beta|^2-|\gamma|^2)=C/(RT)^2, \tag{2}$$

where C is the field attenuation due to loss, R is the constant of the interaction of the atoms with the field, and T is the transverse relaxation time. The number of photons in the resonator (n_0) at a pump level above the threshold $(v>v_0)$ is determined by the equation

$$v(|\beta|^2-|\gamma|^2)\sin^2(RT\sqrt{n_0})=Cn_0\ . \tag{3}$$

Near the threshold $(\delta v=v-v_0\ll v_0)$ the number of photons is simply proportional to the extent to which the pump level exceeds the threshold:

$$n_0=3\delta v/C. \tag{4}$$

The expectation value of the annihilation operator near the threshold is determined by

$$d\langle a\rangle/dt=-\mu_0\langle a\rangle+\epsilon RT\sum_j e^{i\varphi_j}\delta(t-t_j), \tag{5}$$

where j specifies the active atoms, and $\mu_0=C/4n_0$ is the field attenuation constant, which was determined in Refs. 1 and 2. In contrast with Refs. 1 and 2, Eq. (5) for the field contains sources (the second term on the right side), and the field now has a statistically steady-state value which is not zero. These sources are sometimes ignored on the basis that they lead to a sum of many sine waves with random phases, which cancel each other out specifically because their phases are random. It should be noted, however, that (on the one hand) the summation of a large number of sine waves with random phases does not result in a complete extinction of these waves; the field amplitude actually increases in proportion to the square root of the number of sine waves which are being summed (this circumstance has been known since the time of Rayleigh). On the other hand, the second term on the right side of (5) is the only field source in this equation, so it cannot be ignored. It can be shown that the mean square value of the field is, within a factor on the order of unity, $\langle E(t)\rangle_{ms}\approx\epsilon E_0$, where E_0 is the field in the coherent state. In other words, near the threshold, when the superposition states of active atoms are taken into account, the field does not increase to its classical value because of the relation $\epsilon \ll 1$, although the field does become macroscopic.

The active atoms in state (1) have a certain phase φj. If the pump level is sufficiently far above the threshold, it is necessary to deal with the asymmetry in terms of the pump which arises between the atoms which are in phase with the existing field and the atoms which are out of phase with it. The atoms which are out of phase transfer energy to the light faster than the in-phase atoms do. Over a time T, a population difference

$$\Delta|\beta|^2 = \sqrt{2}\epsilon^2 RT\sqrt{n_0} \tag{6}$$

is established between these groups of atoms. Consequently, the admixture of the lower state is greater in the atoms which are out of phase than in those which are in phase. We recall that in lasers the lifetime of the lower level is shorter than that of the upper level. The active atoms which are out of phase will thus stop interacting with the field sooner than the in-phase atoms will. As a result, with the random variations in phase due to the transverse relaxation in the medium (e.g., due to collisions with phonons), a constant flux of in-phase atoms to out-of-phase atoms is formed in lasers:

$$\Delta v \approx \frac{1}{2} v \Delta|\beta|^2 \zeta, \tag{7}$$

where

$$\zeta = (T_\beta - T_\gamma)/(T_\beta + T_\gamma) \tag{8}$$

and T_β and T_γ are the lifetimes of the upper and lower working states.

The rate of pumping by the out-of-phase atoms turns out to be slightly higher than that of the pumping by the in-phase atoms. In other words, the pump level depends on the relative atom-field phase:

$$v(\varphi) \approx v\left(1 + \frac{1}{4}\Delta|\beta|^2 \zeta \sin\varphi\right). \tag{9}$$

The expectation value of the field in the interaction representation is then

$$\langle E \rangle = \alpha \epsilon RT\sqrt{n_0} E_0 \Big[\int_{-\infty}^{t} dt' \frac{(-i)}{2\pi} \int_0^{2\pi} d\varphi\, e^{i\varphi + \mu_0 (t'-t)} \times \left(1 + \frac{1}{4}\Delta|\beta|^2 \zeta \sin\varphi\right) + \text{c.c.} \Big] = \alpha\zeta (2\epsilon^2 n_0)^{3/2} E_0, \tag{10}$$

where $\alpha \approx 1$. The maximum value of the factor ζ is unity (in the case $T_\gamma \ll T_\beta$), and the estimates below are made for specifically this value of ζ, although this factor may be small in some cases.

The square of the expectation value of the field is

$$\langle E\rangle^2 = 8\alpha^2\zeta^2\epsilon^6 n_0^3 E_0^2. \tag{11}$$

Since we have

$$E_0^2 \approx \langle E^2\rangle = Dn_0 = D'\delta v \quad (D' = 3D/C) \tag{12}$$

for a coherent state [we are using (4)], we find

$$\langle E\rangle^2 = D''(\delta v)^4 \quad (D'' = 648D\alpha^2\zeta^2\epsilon^6/C^4). \tag{13}$$

Expression (13), found by perturbation theory in the case $\langle E\rangle^2 \ll \langle E^2\rangle$, changes nature as the pump level increases, since an increase in $\langle E\rangle^2$ causes this quantity to become smaller than $\langle E^2\rangle$, and the relative difference between these quantities decreases. Consequently, there is an inflection point (Fig. 1) on the plot of $\langle E^2\rangle$ versus $\delta v = v - v_0$, at $v = \bar{v}$. This value can be defined as a second threshold value of the pump level (a second threshold), at which the field increases to its classical value, and a coherence arises in the laser in this fashion. Actually, the field increase occurs in a narrow interval of pump levels near the value $v = \bar{v}$, and this second threshold is thus slightly blurred. Since the calculation of $\langle E\rangle^2$ as a function of n_0 and thus δv is based on perturbation theory, it is not possible at this time to determine the position of $\bar{v}$ exactly, so we find the second threshold approximately, as the point at which (12) and (13) intersect (Fig. 1). This approach is equivalent to the case in which the coefficient of E_0 in (10) is unity. The threshold pump level is then found from the condition

$$n_0 = \bar{n}_0 = 1/2\epsilon^2 \quad \text{or} \quad \delta\bar{v} = \bar{v} - v_0 = C/6\epsilon^2. \tag{14}$$

Since ϵ has not yet been determined experimentally, we will offer two estimates: For $\epsilon^2 = 10^{-4}$, we find $n_0 = 2\times 10^3$, which corresponds to a power level $W = 6\times 10^{-9}$ W under typical laser conditions. With $\epsilon^2 = 10^{-10}$, we find $n_0 = 2\times 10^9$ and $W = 6\times 10^{-3}$ W, respectively.

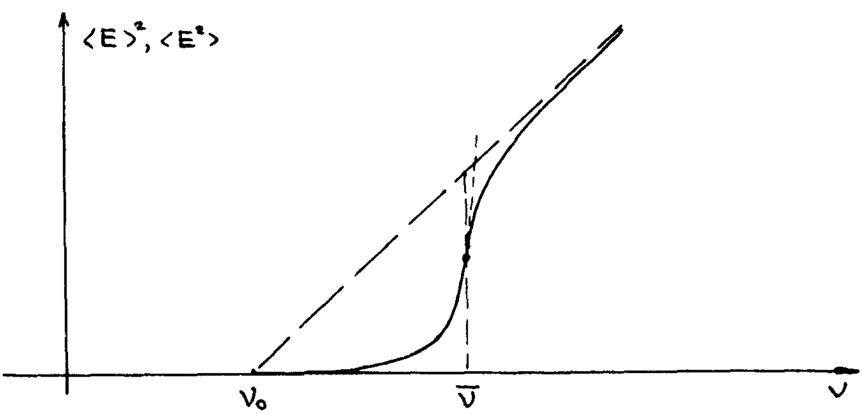

FIG. 1. Plots of $\langle E\rangle^2$ (solid line) and $\langle E^2\rangle$ (dashed line) versus the pump level v. v_0—First threshold; $\bar{v}$—second threshold.

Active atoms in a superposition state are thus sources of field in lasers, and they impart a coherence to the lasers: The active medium simply amplifies the degree of this coherence. The increase of the field to its classical value is determined by the difference between the efficiencies of the in-phase and out-of-phase active atoms.

In the pump interval between the first and second thresholds, the light field is in a substantially quantum-mechanical state,[4] with a field close to zero while the field energy is nonzero. In an experimental study of the vicinity of the threshold, it would be desirable to determine how the square of the expectation value of the field, $\langle E \rangle^2$, depends on the pump level v and to compare this behavior with the behavior of $\langle E^2 \rangle$ as a function of v. These two functional dependences are shown qualitatively in Fig. 1.

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