Second (quantum) threshold of a laser[1)]

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The quantum-mechanical nature of electromagnetic radiation is responsible for the existence of two threshold values of the gain. When the first of these thresholds is exceeded, there is an exponential and incoherent increase in the number of photons in the resonator. When the second threshold is exceeded, a coherent field begins to form.

Technological advances in the laboratory have attracted interest to some subtle effects which occur in a resonant interaction of an electromagnetic field with the medium in a resonator. These effects are known collectively as "in-resonator quantum electrodynamics." Several of these effects were discussed theoretically many years ago,[1–3] but only in recent years have they been confirmed experimentally.[4,5]

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In this letter we discuss an effect related to in-resonator quantum electrodynamics.

One possible approach to the description of the dynamics of lasers starts from the Heisenberg equations for the matter and field operators.[6,7] In the adiabatic approximation, in which the relaxation times for the dynamic variables describing the active medium of the laser are short in comparison with the photon lifetime in the resonator, the dynamics of a laser can be described by a single equation for the photon annihilation operation a:

$$\frac{da}{dt} = (\kappa - \gamma) a - \frac{\kappa}{4n_s}(ac + ca) + F, \tag{1}$$

where κ is the linear gain of the laser, γ is the field attenuation coefficient in the resonator, $c = a^+ a + aa^+$, n_s is the saturation number of photons (this parameter is a measure of the strength of the interaction of the electromagnetic field with the matter), and the operation F represents fluctuation forces.

From (1) we find an equation for the operation $n = a^+ a$:

$$\frac{dn}{dt} = 2(\kappa - \gamma) n - 2\kappa \frac{n^2}{n_s} + F^+ a + F a^+. \tag{2}$$

We see that if $\kappa > \gamma$, the average number of photons in the resonator will grow with time; at the beginning of the process, the growth will be exponential, with $\kappa > \gamma$. This is the usual threshold condition for a laser.

Expressing the operator c in terms of the photon number operator, we can put Eq. (1) in the form

$$\frac{da}{dt} = (\kappa - \gamma) a - \frac{\kappa}{n_s}(n + 1) a + F. \tag{3}$$

It can be seen from this equation that under the conditions $\gamma < \kappa < \gamma n_s/(n_s - 1)$ the coherent field described by the expectation value of the operator a does not form, since the expectation value of the fluctuation operator F is zero. The meaning is that the field generated by the laser in this case has statistics different from the statistics of a coherent field.

A laser thus has two thresholds: $\kappa_1 = \gamma$ and $\kappa_2 = \gamma n_s/(n_s - 1)$. The second threshold is of a purely quantum-mechanical nature: It stems from the circumstance that the operators a^+ and a do not commute.

In an ordinary laser the condition $n_s \gg 1$ holds, so there are essentially no differences between κ_1 and κ_2. This effect, however, may be significant for a laser excited by a small number of atoms.[4] At $n_s = 10$, for example, the difference between κ_1 and κ_2 reaches 10%, quite amenable to experimental observation. For a beam laser we would have $n_s = \hbar(8\pi \times \omega\mu^2\tau^2)^{-1}$, where μ is the dipole moment of the resonant transition in the atom, ω is the resonant frequency of this transition, and τ is the transit time of an atom through the resonator. With $\tau = 10^{-4} - 10^{-6}$ s, the value of n_s for a Rydberg transition with the principal quantum number on the order of 10 would be no more than 10.

Observing this two-threshold regime would require that the resonator walls be at a low temperature, so that the average thermal energy of the thermal radiation in the resonator would be less than $\hbar\omega$. A beam of atoms in the lower of two resonant levels can cool a resonator by resonantly absorbing thermal radiation.

Bykov and Shepelev[8] have raised the question of whether incoherent light can be generated by a laser, but they asserted that a laser would generate a coherent field only if the pump created not only a population inversion but also an initial polarization of the active particles. In the absence of an initial polarization, according to Ref. 8, a laser would always generate incoherent light. However, Eq. (1) shows that under the condition $\kappa > \kappa_2$ a coherent field ($\langle a \rangle \neq 0$) is produced without any initial polarization. This point was mentioned even in Ref. 9.

[1)]A report made on 12 December 1989 in Zvenigorod at an expanded session of the "Laser Phosphors" Section of the Scientific Council of the Academy of Sciences of the USSR on "Luminescence and Developments in its Applications in the Economy."

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