Substantially nonlinear new surface modes at plasma–vacuum interface

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The properties of some substantially nonlinear new surface modes with even and odd numbers of discontinuities in the dielectric constant are studied. These modes have an anomalous dispersion and a normal dispersion, respectively, at frequencies near the plasma resonance.

1. An important aspect of media having an interface is the possible propagation of surface electromagnetic waves in them, at a phase velocity which does not exceed the velocity of light in the medium. The rapid development of nonlinear electrodynamics of surfaces has made it necessary to study the dispersion characteristics of such waves at elevated power levels, at which the nonlinearity of the constitutive equations for the field must be taken into account. The derivation of a nonlinear theory of surface electromagnetic waves dates back to the study by Alanakyan[1] of the nonlinear dispersion properties of p-polarized surface waves. Litvak and Mironov[2] showed that it was possible to construct nonlinear s-polarized surface electromagnetic waves (see also the review by Boardman and Egan[3]). A characteristic feature of p-polarized surface electromagnetic waves is the possible appearance of a singularity in the field near the plasma resonance. In a semi-infinite homogeneous plasma, a singularity of this sort may appear at frequencies in which linear surface electromagnetic waves exist $\omega<\omega_p/\sqrt{2}$, where ω_p is the plasma frequency) at substantial power levels of the surface electromagnetic waves. Below we show that at frequencies near the plasma resonance ($\omega\approx\omega_p$), where the nonlinear properties of the plasma are manifested even at low power levels, some new and substantially nonlinear surface modes appear. We derive the threshold values of the field above which such modes are realized. We derive and study nonlinear dispersion relations for them.

2. We consider a semi-infinite ($z\geqslant 0$) plasma bordering a vacuum ($z<0$). We seek steady-state solutions of the nonlinear field equations for surface waves which have a frequency ω and a wave vector k [$\sim\exp(-i(\omega t-kx))$]:

$$\frac{\partial B_y}{\partial z}=i\frac{\omega}{c}\epsilon E_x,\ kB_y=-\frac{\omega}{c}\epsilon E_z,\ \frac{\partial E_x}{\partial z}-ikE_z=i\frac{\omega}{c}B_y, \tag{1}$$

where E_x, E_z, $B_y\equiv B$ are the field amplitudes, which satisfy the boundary conditions that the tangential components of the fields are continuous, $\{E_x\}=0$ and $\{B\}=0$, at the interfaces. Equations (1) have a first integral:

$$(1-\epsilon_0)I-\epsilon+\epsilon_0-\frac{2\eta^2-\epsilon}{\epsilon}B^2=\mathcal{H}, \tag{2}$$

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where $\mathscr{H}$ is a constant of integration, $\epsilon_0 = 1 - \omega_p^2/\omega^2$ is the linear dielectric constant, and $\epsilon = \epsilon(I)$ is the nonlinear dielectric constant, which depends on the wave intensity $I = |\mathbf{E}|^2/E_p^2$ $(E_p^2 = 4mT\omega_p^2/e^2)$ and satisfies the relation

$$\epsilon(1-\epsilon_0)I = (B')^2 + \eta^2 B^2. \qquad (3)$$

Below we consider the case of a ponderomotive nonlinearity of a plasma with saturation, which is characterized by the following intensity dependence of the dielectric constant:

$$\epsilon = 1-(1-\epsilon_0)\exp[-I(1-\epsilon_0)]. \qquad (4)$$

We assume $\epsilon_0 < 0$.

A qualitative analysis of system of equations (2)–(4) can be carried out in the (B',B) phase plane by analogy with Ref. 4, where the propagation of strong, ionizing surface electromagnetic waves in a medium with $\epsilon_0 > 0$ was studied.

It follows from an analysis that near the plasma frequency (Langmuir frequency) one can construct solutions which correspond to a retarded surface electromagnetic wave, if one assumes that there can be discontinuous plasma-field distributions[5] which correspond to our approximation of ignoring the spatial dispersion of the waves. The lowest threshold field corresponds to solutions with two discontinuities, which form a density well in which electromagnetic field is trapped. The dispersion relation for these modes is

$$\omega = \omega_p\left(1 - \frac{3}{4}I_i - I_i^2 + \frac{I_i^2}{8}\frac{\omega_p^2}{k^2c^2}\right). \qquad (5)$$

This relation corresponds to surface electromagnetic waves with an anomalous dispersion. The propagation of this mode is possible if the wave field, alternating at the frequency ω, at the plasma–vacuum interface, i.e., I_i, lies in the following interval:

$$\frac{2}{3}\left(1 - \frac{\omega}{\omega_p}\right)\left(1.278\frac{\omega}{\omega_p} - 0.278\right) \leqslant I_i \leqslant \frac{4}{3}\left(1 - \frac{\omega}{\omega_p}\right)\left(1.54\frac{\omega}{\omega_p} - 0.54\right). \qquad (6)$$

After the "gap" along the scale of the field I_i, in which the propagation of surface electromagnetic waves is not possible, is passed, there are solutions with a single discontinuity under the following conditions:

$$2.523\left(1 - \frac{\omega}{\omega_p}\right)\left(1.384\frac{\omega}{\omega_p} - 0.384\right) \leqslant I_i \leqslant 2.667\left(1 - \frac{\omega}{\omega_p}\right)\left(1.361\frac{\omega}{\omega_p} - 0.361\right). \qquad (7)$$

They correspond to surface electromagnetic waves with a normal dispersion:

$$\omega = \omega_p\left(1 - 0.396 I_i - 0.206 I_i^2 - 0.012\frac{I_i^2}{k^2c^2}\omega_p^2\right). \qquad (8)$$

Dispersion relations (5) and (8) describe surface electromagnetic waves with an arbitrary even number of jumps and an odd number of jumps, respectively. It can be assumed on the basis of energy considerations, however, that the lowest-index modes, with two or one jump, are realized.

3. The physical reason for the appearance of substantially nonlinear new branches of surface electromagnetic waves lies in a ponderomotive modification of the dielectric constant of the plasma. The wave branches which have been found here can be interpreted in the linear theory of waves in a layered structure, which corresponds to a self-consistent plasma-field distribution. The possibility of a realization of solutions near the plasma resonance for surface electromagnetic waves with density discontinuities was demonstrated by numerical calculation in Ref. 7 in a study of a nonlinear resonant absorption of electromagnetic radiation in a slightly inhomogeneous plasma at higher power levels. The results found in this study are evidently not tied to any specific model of a nonlinear medium and can easily be generalized to the case of arbitrary nonlinear media in which there can be a mutual conversion of various wave modes.

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