Superconducting quantum interferometer based on metal-oxide ceramic in hysteresis-free operation at 77 K

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It has been shown experimentally that an rf SQUID can operate even if the energy of the Josephson junction does not exceed the thermal energy. A SQUID of ceramic $YBa_2Cu_3O_x$, which has been developed, is capable of hysteresis-free operation in liquid nitrogen and has a noise level less than $3\times10^{-4}\,\Phi_0\sqrt{\mathrm{Hz}}$. The parameters of this SQUID have been determined. The roles played by the low-frequency and high-frequency components of thermodynamic fluctuations of the magnetic flux in the SQUID are evaluated on the basis of a resistive model for the Josephson junction.

In a recent review on SQUID magnetometers,[1] fundamental limitations on the possibility of observing a response from a device were reported. One of these restrictions was derived under the assumption that the energy of the Josephson junction must be higher than the thermal energy, i.e., under the assumption $I_C\Phi_0/2\pi > k_B T$. Here $\Phi_0=\pi h/e\approx 2\times10^{-15}$ *Wb* is the quantum of magnetic flud, and I_c is the critical current of the weak link. This condition is sometimes written in terms of the thermal-fluctuation parameter $\gamma=2\pi k_B T/I_c\Phi_0$:

$$\gamma<1. \tag{1}$$

At $T=77$ K, we accordingly have the restriction $I_c>2\pi k_B T/\Phi_0\approx 3.2$ μA.

The parameter which basically determines the operating regime of a single-junction quantum interferometer is its dimensionless inductance $l=2\pi L_s I_c/\Phi_0$, where L_s is the inductance of the ring of the detector. The value $l=1$ is the boundary between hysteresis-free operation ($l<1$) and hysteretic operation ($l>1$) of the SQUID. In rf-SQUID detectors of Zimmerman design, the condition $L_s>10^{-10}$ H usually holds; from this condition and the estimate of I_c found from (1) we find $l>1$. This result means that it would not be possible to fabricate a Zimmerman detector of an rf SQUID capable of hysteresis-free operation at $T=77$ K.

Our purposes in the present study were to experimentally prove that hysteresis-free operation of an rf SQUID can be achieved in liquid nitrogen and to offer a qualitative explanation of the results.

In the experiments we used cylindrical detectors of $YBa_2Cu_3O_x$, 4–6 mm long, with an aperture of 0.1 mm. The wall thickness was 0.5–1 mm. The cylinder was cut parallel to its axis, and bridges with typical dimensions of 30–50 μm were formed in the cuts. A coupling coil, $\approx$0.9 mm in diameter, was placed in an aperture. The coil was connected to an oscillator circuit with a resonant frequency of 20–30 MHz and an unloaded quality factor $Q \approx 50$ at 77 K or $Q \approx 100$ at 4.2 K. We recorded the alternating voltage $U_\sim$ as a function of the pump current $I_\sim$ (current-voltage characteristics) and $U_\sim$ as a function of the direct current through the coupling coil, $I_=$, at a fixed $I_\sim$ (the signal characteristic). The measurement arrangement is described in more detail in Ref. 2. The measurements were carried out in liquid helium in superconducting shields and in liquid nitrogen in multilayer shields of magnetically soft materials.

Figure 1 shows an rf current-voltage characteristic (curve 1) and a signal characteristic (curve 2) written at 4.2 K. The results obtained in liquid helium were analyzed for the purpose of determining the "geometric" parameters of the SQUID: L_s and the coefficient (k) of the coupling of the resonant circuit with the detector. This coupling coefficient was found from the shift of the resonant frequency of the oscillator circuit, $k = \sqrt{(\omega_0 + \omega')(\omega' - \omega_0)}/\omega_0 = 0.48$. Here ω_0 and ω' are the resonant frequencies of respectively the unloaded circuit and the circuit with the interferometer, measured at a low pump level, on the linear region of the rf current-voltage characteristic. The mutual induction $M = \Phi_0/\Delta I_= \approx 10^{-8}$ H was determined from the period of the signal characteristic, $\Delta I_= = 0.2\ \mu$A (curve 2 in Fig. 1). Finding the inductance of the resonant circuit from its impedance, $L_K \approx 4 \times 10^{-7}$ H, and using $M = k\sqrt{L_s L_k}$, we calculate $L_s \approx 7 \times 10^{-10}$ H. From the rf current-voltage characteristic (under the assumption that the critical current is reached in the interferometer at $U_\sim \approx 50\ \mu$V across the circuit), we find $I_c \approx 10\ \mu$A.

We place a holder with the sample in liquid nitrogen. Figure 2 shows rf current-voltage characteristics of the interferometer at 77 K. These characteristics have been

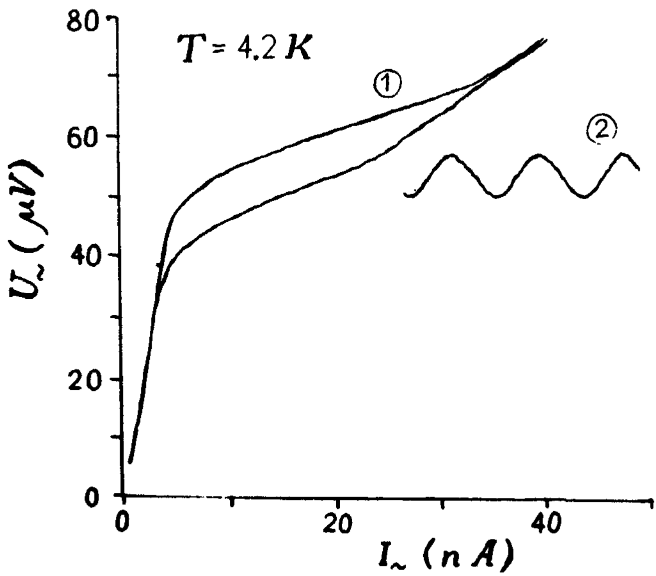

FIG. 1. 1—An rf current-voltage characteristic of a SQUID; 2—signal characteristic at 4.2 K. The period of the signal characteristic is 0.2 μA.

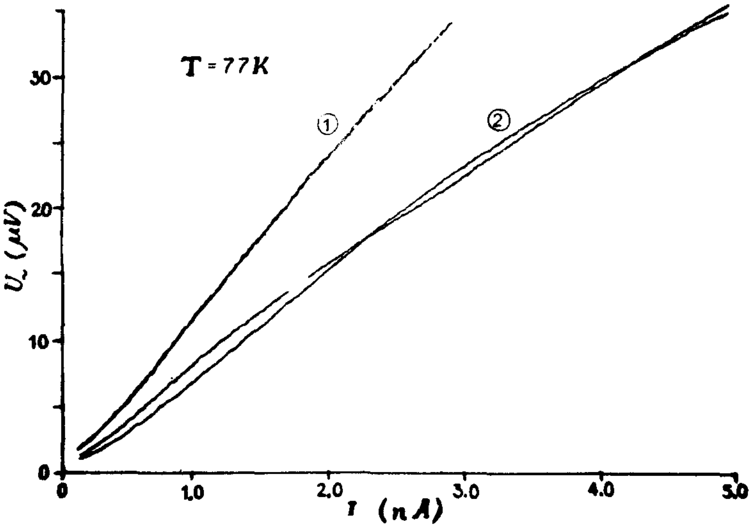

FIG. 2. An rf current-voltage characteristic of a SQUID at the resonant frequency (1) and one at a frequency deviation $\omega/\omega' - 1 = 1.3\times 10^{-2}$ (2), recorded at values $n\Phi_0$ and $(n+1/2)\Phi_0$ of the magnetic flux in the detector at 77 K.

written for an integer number and a half-integer number of flux quanta in the SQUID at the resonant frequency ω' (curve 1) and at a frequency deviation $\omega/\omega' - 1 = 1.3\times 10^{-2}$ (curve 2). These curves are characteristic of the hysteresis-free operation of the device.[3]

In hysteresis-free operation, because of the parametric inductance of the Josephson junction, ω' is a function of the magnetic flux in the aperture of the detector. In an effort to determine the reasons for the appearance of a response of the SQUID to a change in the magnetic field, we thus recorded a family of signal characteristics at various frequency deviations, and we reconstructed the amplitude-frequency characteristics of the device (Fig. 3). It can be seen from Fig. 3 that the response of the device is related to the change in ω'. This result proves conclusively that hysteresis-free operation has been achieved.

The SQUID is thus capable of operation when condition (1) is violated. To explain the results, we work from the resistive model for Josephson junctions.[4] The Josephson current $I = I_c \sin(2\pi\Phi/\Phi_0)$ is determined by the magnetic flux Φ in the detector ring. If the noisy magnetic flux caused by the fluctuation current does not exceed $\Phi_0/2$, the response will therefore be observed despite a violation of (1). The mean square value of the noise flux in a frequency band $\delta\omega/2\pi$ is, according to the resistive model,[4]

$$\langle \delta\Phi^2 \rangle = 2k_B T L_s^2 \delta\omega / \pi R(1+q^2), \tag{2}$$

where $q = \omega L_s / R$, and R is the normal resistance of the weak link. An integration over all frequencies gives us the total noise flux, $\langle \Phi_n^2 \rangle$. The condition $\langle \Phi_n^2 \rangle \leqslant (\Phi_0/2)^2$ then becomes

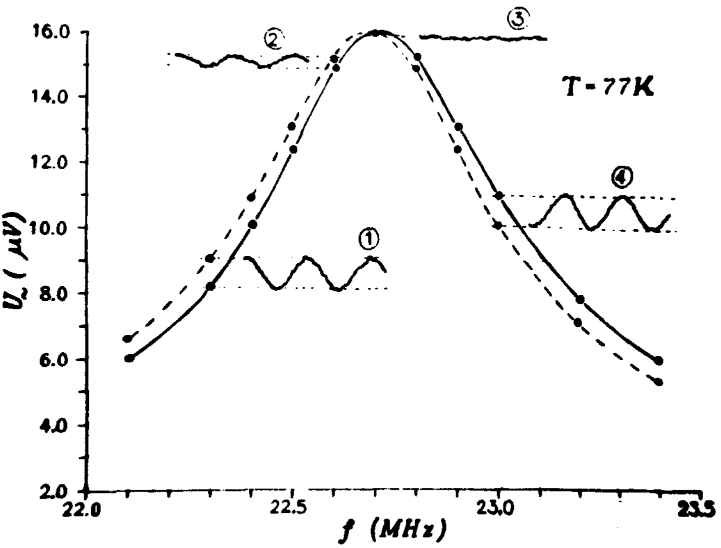

FIG. 3. Amplitude-frequency characteristics for an integer number (solid line) and a half-integer number (dashed line) of flux quanta Φ_0 in the SQUID ring, reconstructed from a family of signal characteristics, along with examples of signal characteristics recorded at several pump frequencies at 77 K: 1—22.3; 2—22.6; 3—22.7; 4—23.0 MHz.

$$\gamma l/\pi^2 \leqslant 1 \, . \tag{3}$$

If we define the noise inductance as $L_n = \Phi_0^2/4k_B T$, we see that (3) is equivalent to $L_s \leqslant L_n$. Since we have $L_s \approx 7\times 10^{-10}$ H for our SQUID, and since at 77 K we have $L_n \approx 10^{-9}$ H, we find $\gamma l/\pi^2 = 0.7$. In other words, the interferometer operates in the face of large thermodynamic fluctuations. Our next purpose was to find the low-frequency ($\omega < \omega_p$, where ω_p is the pump frequency) component of the interferometer noise. For numerical estimates we determined the quality factor of the SQUID detector at the pump frequency, $q_p = \omega_p L_s/R$, from the relation $1/Q' - 1/Q \approx k^2 q_n$, where Q' is the quality factor of the circuit with the interferometer. In our case, Q' differs from Q by no more than 5%, so we have $q_p \leqslant 4\times 10^{-3}$. Integrating from 0 to ω_p in (2), and ignoring q^2 in the denominator, since we have $q < q_p$ at $\omega < \omega_p$, we find the square of the noise flux, normalized to $(\Phi_0/2)^2$, in the band to ω_p:

$$\frac{\langle \Phi_n^2 \rangle}{(\Phi_0/2)^2} = \frac{2q_p}{\pi}\,\frac{\gamma l}{\pi^2} \leqslant 2\cdot 10^{-3} , \tag{4}$$

This result corresponds to a noise level $\sim 10^{-5}\,\Phi_0\sqrt{\mathrm{Hz}}$. The role of the detector noise at frequencies $\omega > \omega_p$ reduces to one of averaging the interferometer phase $\varphi = 2\pi\Phi/\Phi_0$ and thus reducing the conversion coefficient $dU_{\sim}/d\Phi$. As a result, the effective contribution of the circuit noise and the amplifier noise is increased. These noise components apparently also determine the measured sensitivity of the device, 3×10^{-4}

$\Phi_0/\sqrt{\text{Hz}}$. These conclusions are supported by the following considerations. We can estimate l and I_c of the SQUID from the expression $k^2 l \approx a(\omega_1 - \omega_2)/2\omega_r J_1(a)$, which was derived in Ref. 5, in the course of an analysis of hysteresis-free operation. Here ω_r is the resonant frequency of the circuit with the detector when the dimensionless amplitude of the oscillations in the circuit is $a = 2\pi I_{\sim} M/\Phi_0 \gg 1$, $J_1(a)$ is the Bessel function of the first kind, and ω_1 and ω_2 are the resonant frequencies for the cases of an integer number and a half-integer number of flux quanta in the interferometer, measured at a pump level $a < 2\pi$. We thus find $l \approx 0.03$ and $I_c \approx 15$ nA. These estimates are not plausible, since the transition temperature of the ceramic is ~ 90 K, and we would hardly expect the critical current to decrease by three orders of magnitude as the temperature is raised from 4.2 K to 77 K. We believe that the values of l and I_c are too low, since the rf noise "washes out" the measured quantity $\omega_1 - \omega_2$.

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